What is the BrainTrax System?

The BrainTrax System is a learning tool that delivers high-quality math-related content through a web-browser. This information is displayed via a visual index. A visual index is a graphical means of showing how different concepts relate to each other. In the Algebra Brain, for example, we show all the different components of algebra that are related to the concept of Polynomial Functions.

The BrainTrax System contains features not found anywhere else on the internet. We include three levels of content, appropriate for eighth grade on up through college. We also include interactive examples, real-world example problems, and an Interactive Example and Testing System (IETS) so teachers can monitor the progress of their students.

Why do I care?

With technology quickly growing beyond all human comprehension, today's students need greater access to technology to improve their education. It is in the best interests of educators to meet the demands of tomorrow by introducing students to the technology of the future, i.e. the internet. The BrainTrax System is designed to be user-friendly to both students and teachers. Typical students will find that they retain the information more effectively than simply reading a textbook. We expect teachers will find that students who are actively engaged in learning from the BrainTrax System will be better prepared for college-level courses when they graduate from high school.

Can I use this?

Nearly anyone can use the BrainTrax system. It is open to the public and available free of charge at this time in most schools. The BrainTrax System is not available to students and instructors of four-year colleges and universities (with the exception of UMR). Several middle and high school teachers are helping us develop tools that will allow other instructors to integrate the BrainTrax System into the classroom environment. The BrainTrax System uses a very intuitive interface and requires a short period of adjustment. Students exposed to the BrainTrax System become expert users with a couple of days or even a few hours. Teachers can also extend the system by incorporating their own tests and examples through the IETS.

Is it expensive to implement?

As we already stated, it is FREE at this time, except for other colleges and universities. Teachers who would like more information about how they can get involved should contact Mark Bookout (contact information below). Teachers may want to attend workshops, for a nominal fee, in order to get the training they need to implement the BrainTrax System in the classroom, but this is by no means a requirement for using the System.

How do I view the BrainTrax System?

All you need to see the visual index is a good browser. Microsoft Internet Explorer 5.0 or higher seems to be the best one on the market for viewing our indexes. Currently, IE5.0+ is the only browser we officially support. Netscape Navigator is not officially supported, as it doesn't handle portions of the BrainTrax System very well. We do not in any way, shape or form, support the BrainTrax System on a Macintosh.

How is the BrainTrax Algebra System any different than a hundred other websites?
No other site that we know of offers the sheer comprehensive knowledge base that we are offering. The BrainTrax Algebra Brain alone contains nearly 400 pages of information, with detailed explanations, clear illustrations of concepts, and examples. In addition, we have feedback mechanisms to guide students in solving problems interactively. We include an Interactive Example and Testing System so that students can take a test for a grade. It also has feedback mechanisms to guide students.

The Visual Index itself is a feature unique to the BrainTrax System. It shows students how the different concepts are related to each other. By navigating the Visual Index, students reinforce the connections between concepts.

**Why should I go to all that trouble of implementing the BrainTrax System?**

The BrainTrax System has numerous benefits. First and foremost, student success rate should increase as they become accustomed to its intuitive interface. We have already seen marked results from students who have used the Calculus I Brain. By increasing student success rate at the junior high and high school level, we will hopefully also be increasing their chances at succeeding in college, where the work becomes much, much more difficult.

**What can I use the BrainTrax System for?**

Currently the BrainTrax System is designed as a mathematical learning tool, supporting algebra, trig, even calculus. However, there are numerous other applications. One model is a classroom database containing all the information about a class, such as test scores, syllabi, projects, and more. Although we use it for math, there is no reason why a similar index could not be built around a science class, such as physics or chemistry. All of the features included in the Algebra Brain can be adapted to fit the needs of almost any subject. It is mostly a matter of developing the content.

**How can I learn more about the BrainTrax System?**

The University of Missouri – Rolla has successfully implemented the technology at a basic level in the classroom and has plans for more developed implementation in the future. UMR has also provided access to the BrainTrax System to junior high schools, high schools and grade schools for use in the classroom.

For more information about the BrainTrax System contact:

**Attn: BrainTrax Development**
UMR Computer and Information Services
104 Computer Science Building
1870 Miner Circle
Rolla, MO  65409–1110

Phone:  (573) 341–4841
Fax:   (573) 341–4216
Email: braintrax@umr.edu
As a new BrainTrax user, we want you to be sure you know of all the available features.

This is what the new BrainTrax Algebra Brain looks like:

Notice that the browser shown does NOT have a back button displayed. That is because the brain must be used as the navigational tool. Click on the top thoughts to go back to the root of the matrix. Clicking the lower (child) thoughts moves you closer to the leaf nodes of the matrix. There are no more than 5 levels from top to bottom. There are approximately 130 thoughts, and about 120 separate concepts are covered.

The BrainTrax display has three primary regions, the Upper Region, the narrow Middle Region, and the Lower, or content, Region.

**The Upper Region**

The upper region is the Brain itself. It is used for navigation around and through the content contained with the BrainTrax Algebra Brain.

In addition to the brain itself, in this region you will also find a search box (it is the white box in the lower right hand corner of the blue part). Typing in this box will search the thought names for a partial match. Searching will begin as soon as you type. Note that this does not search the content of the thought, only the thought name.
The Middle Region

The middle portion, or Title Window, may contain a variety of discrete items, each of which has some significance for you, the user. Let's look at each of these in turn.

Title: this is a label that identifies the subject matter of the page you are reading. In our example here, it is called "Basic Rules of Algebra".

Level Tabs: Each content page within the algebra brain has 3 levels (where the material permits). These are Basic, Intermediate, and Advanced.

- The Basic level is aimed at students with an 8th grade comprehension of mathematics. These pages have a purple border.
- The Intermediate level is aimed at students with a 10th grade level of comprehension. These pages have a Green border.
- The Advanced tab is aimed at students ready for algebra at a collegiate level, such as is taught at UMR. These pages have a blue border, and are the default level displayed within the brain at this time.

Clicking one of these tabs will display the appropriate level of information for the subject listed in the "Title Window". In addition to the border color, (purple, green, or blue), the selected tab will be highlighted, or appear brighter when it is active.

Real World Example Tabs: Each advanced level page has one or more real world examples to demonstrate the use of the mathematical concept. Intermediate and Basic levels may have this feature in the future. Teachers who wish to assist the BrainTrax team in creating suitable examples should discuss this with the BrainTrax office.

The Descending Grades Button: Clicking this button will launch an Interactive Example and Testing System (IETS) session for the displayed content. See provided reference material on the BrainTrax IETS for further detail.

The IETS session will be specific to both a given thought and a content level.

Only authorized students will be allowed to enter the IETS to participate in this activity.

The Lower Region

Student Guide Buttons: Each page will have 3 or more buttons that will, when moused over, display important information for the student. At the top of each page will be "Goals", "Terms", and "MathTrax".

- Goals – Mousing over this button will reveal the learning goal for this page
- Terms – Mousing over this button will reveal a list of terms that are used on this page, and their definitions
- MathTrax – Mousing over this button will reveal a math factoid that is traditionally not provided in the classroom.
- Example – Mousing over this button will reveal a description of what the student should learn from this example.

Note that content displayed by these buttons may be different for different levels within a single thought.

Scroll Bars: Notice that the lower section of the example picture shows scroll bars. Use these to see the remainder of the page displayed.
The following buttons may appear in the colored border on the left side of the lower region.

**The e Button:** Clicking this button will jump to a related static example within the three level pages. Mousing over the button will tell the user where they will be transferred if they click the button.

**The i Button:** This button conveys additional information. It is used to tell the student where they may find more basic or advanced material about the specific component next to the button. Clicking the button will transfer the student to that location on another level's page.

**The Speaker Button:** Clicking this button will activate an audio stream that discusses the material directly opposite the button. For text material, the audio stream will read the text. For images, it will describe what the student is seeing, and what they should notice about the image. This feature is available on basic level pages only.

**The ? Button:** Clicking this button will spawn a new window with a question/answer activity for the student. Questions are generally conceptual in nature. Two answers are displayed, along with an explanation after the student has selected his/her answer.

**The x Button:** Clicking this button will spawn a new window with a dialog to solve a problem. The student will be asked at each step what they should do to solve the problem. (est. available Oct 2001)

**The Movie Button:** Clicking this button will launch a short movie detailing a specific math operation or task. The movie will appear in a new window. Not all pages will have embedded video.

**Contact Information**

You can reach UMR's BrainTrax developers here:

**BrainTrax Development**
Attn: Mark Bookout
104 Computing Services
1870 Miner Circle
Rolla, Missouri 65409

(573) 341 4841

Or via email to braintrax@umr.edu
Algebra Glossary (jump) – Several definitions of common terms used throughout Algebra
Algebra Alphabet – A list of how each letter of the alphabet is used throughout Algebra

Fundamentals of Algebra (jump) – The basics needed to fully understand what Algebra is all about
Course Content (jump) – The 5 basic concepts covered in all Algebra courses, from 8th grade to 13th grade
Algebraic Expressions – The expression is one of the basic units used in Algebra, is part of an algebraic function
Basic Rules of Algebra – Covers many of the fundamental properties exhibited by different operators in Algebra

Properties of Equality – What equality really means
Properties of Exponents – How to manipulate exponents to simplify algebraic expressions
Exponential Functions (jump) – Basic definition of the exponential function
Properties of Fractions – How to manipulate fractions to simplify algebraic expressions
Rational Functions (jump) – One polynomial function divided by another
Properties of Inequalities – A list of common properties that are obeyed by inequalities
Linear Inequalities (jump) – How to solve an inequality instead of an equation
Properties of Zero – Fundamentally, very, very important in order to fully understand Algebra

Cartesian Plane – A brief review of the Cartesian Plane, including the Distance and Midpoint Formulas
Radicals – An introduction to square roots, cube roots, and other roots
Properties of Radicals – How to manipulate radicals to simplify algebraic expressions
Real Numbers – Introduction to different number sets used throughout Algebra
Absolute Value – Introduction to the absolute value operator and what it really means
Ordering Real Numbers – How real numbers relate to each other with respect to the origin

Functions – Introduction to one of the fundamental concepts in Algebra
Function Terminology (jump) – A list of common terms associated with functions
Inverse Functions – Interchanging the domain and range of a function
Composition of Functions (jump) – A function of a function
Finding the Inverse – How to find the inverse of a function
Horizontal Line Test – The test which determines if a function has an inverse

Polynomial Functions – Functions involving a sum of powers of x
Fundamental Theorem of Algebra (jump) – The number of roots of a polynomial function equals the highest power of x
Complex Numbers – Numbers the complex plane, of which the real numbers are a subset
Polynomial Operations – How to manipulate polynomials
Adding/Subtracting – How to add and subtract two (or more) polynomials
Multiplying Polynomials – How to multiply two (or more) polynomials
Special Product Patterns (jump) – A list of commonly occurring products that occur in Algebra
Polynomial Division – How to divide two polynomials
Synthetic Division – Technique used to divide polynomials of a particular form
Factor Theorem – A polynomial f(x) has a factor (x – k) if and only if f(k) = 0
Remainder Theorem – If a polynomial f(x) is divided by (x – k), then the remainder is r = f(k)
Real Zeros – The real solutions of a polynomial equations (as opposed to the complex solutions)
Descartes's Rule of Signs – How to determine the number of positive and negative roots of a polynomial equation
Rational Zero Test – How to find possible roots of a polynomial function with integer coefficients
Special Factoring Patterns – A list of commonly occurring factoring patterns that occur in Algebra
Special Product Patterns – A list of commonly occurring products that occur in Algebra
Multiplying Polynomials (jump) – How to multiply two (or more) polynomials
Rational Functions – One polynomial function divided by another
Conic Sections (jump) – A look at some practical examples of rational functions
Polynomial Division (jump) – Intimately tied with rational functions
Properties of Fractions (jump) – Necessary for understanding how rational functions work
Asymptotes – What they are and how they are used to help sketch rational functions
Partial Fractions – How to convert a rational function into a sum of two fractions
  Distinct Linear Factors – Denominator of rational function is composed of distinct linear factors
  Distinct Quadratic Factors – Denominator of rational function is composed of irreducible quadratic factors
  Mixed Factors – Denominator of rational function contains both linear and quadratic factors (distinct and/or repeating)
  Repeated Linear Factors – Linear factors that repeat themselves in the denominator of a rational function
  Repeated Quadratic Factors – Irreducible repeating quadratic factors in the denominator of a rational function
Sketching Rational Functions – How to sketch the graph of a rational function
Transcendental Functions – A category of functions that includes both exponential and logarithmic functions
  Exponential Functions – Basic definition of the exponential function
  Properties of Exponents (jump) – The various rules governing the behavior of exponents
  Exponential / Logarithmic Equations – How to use logarithms to solve mathematical equations
    Logarithmic/Exponential Models – Applications of logarithmic/exponential functions
    Exponential Growth and Decay – Very common application of logs
    Gaussian – Model which produces the bell-shaped curve used in statistical analysis
    Logarithmic – Used in a wide variety of applications including earthquakes, sound, and time of death
    Logistics Growth – Model used to accurately represent population growth in an environment
Logarithmic Functions – The inverse of an exponential function
  Properties of Logarithms (jump) – Various rules governing behavior of logarithmic functions
  Exponential / Logarithmic Equations – How to use logarithms to solve mathematical equations
    Logarithmic/Exponential Models – Applications of logarithmic/exponential functions
    Exponential Growth and Decay – Very common application of logs
    Gaussian – Model which produces the bell-shaped curve used in statistical analysis
    Logarithmic – Used in a wide variety of applications including earthquakes, sound, and time of death
    Logistics Growth – Model used to accurately represent population growth in an environment
Natural Base e – Definition of logarithm of base e
Translations and Combinations – Moving graphs around the coordinate plane
  Arithmetic Combinations – How to add, subtract, multiply, and divide two functions
  Composition of Functions – A function of a function
  Reflections in Coordinate Axes – How to reflect a function around either the vertical or horizontal axis
  Vertical and Horizontal Shifts – How to move a function vertically and horizontally around a graph
**Quadratics** – Second-degree polynomial functions

**Conic Sections** – Classification of conic sections by the discriminant of a quadratic function
- **Circles** – Definition of a circle and its related equation
- **Ellipses** – Definition of ellipse and its related equation
- **Hyperbolas** – Definition of hyperbola and its related equation
- **Parabolas** – Definition of a parabola and its related equation

**Translations of Conics** – Moving the graph of a conic section around in the plane
- **Circle Translation** – How to move a circle around in the plane
- **Ellipse Translation** – How to move an ellipse around in the plane
- **Hyperbola Translation** – How to move a hyperbola around in the plane
- **Parabola Translation** – How to move a parabola around in the plane

**Quadratic Equations** – One of the most important applications of polynomial functions
1. **Factoring** – Factor the quadratic, then set each factor equal to zero to find the roots
2. **Extracting Square Roots** – Extracting a square root from a quadratic equation
3. **Completing the Square** – Transforming a quadratic into a perfect square
4. **Quadratic Formula** – The granddaddy of all methods of solving a quadratic when nothing else works

**Linear Equations** – Equations that involve variables raised to the first power only

**Linear Inequalities** – How to solve an inequality instead of an equation
- **Absolute Value Inequalities** – Inequalities involving absolute value operator
- **Polynomial Inequalities** – How to find test intervals for solving polynomial inequalities
- **Rational Inequalities** – Solving inequalities that involve one polynomial divided by another

**Properties of Inequalities (jump)** – A list of common properties that are obeyed by inequalities

**Linear Systems** – How to solve more than one linear equation at a time, often involving two or more variables

**Linear Systems in a Matrix (jump)** – Relationship between linear equations and matrices
- **Graphical Approach** – How to solve a linear system from a graph of the equations
- **Graphical Interpretation of Systems (jump)** – What linear systems have to do with the real world
- **Method of Elimination** – How to solve a linear system of equations by eliminating a variable
- **Multivariable Linear Systems** – Transformation of a linear system into row-echelon form
- **Substitution Method** – Summary of how to solve a system of linear equations using back substitution

**Systems of Inequalities** – Solving and graphing systems of linear inequalities

**Lines and Slope** – Various formulas involving lines and their slopes
- **Equations of Lines** – Several different ways of writing equations of lines
- **Parallel and Perpendicular Lines** – Basic definitions
- **Point-Slope Form of a Line** – One form of an equation representing a line
- **Slope-Intercept Form of a Line** – One form of an equation representing a line

**Matrices and Determinants** – Basic definition of a matrix

**Determinants** – A special operation performed on a matrix
- **Properties of Determinants (jump)** – A list of properties that we can use to simplify the process of finding determinants

**Applications of Determinants** – Several useful ways to apply matrices
- **Area of Triangle** – How to find the area of a triangle with the given vertices using determinants
- **Cramer’s Rule** – Solving a system of linear equations using determinants
- **Lines in Plane** – How to tell if three points lie on the same line and a method for finding the equation of a line

**Triangular Matrix** – A handy form of matrix used to quickly solve linear systems of equations

**Inverse of a Square Matrix** – Analogous to the inverse of a function
- **Finding Matrix Inverse** – How to find the inverse of a matrix
- **Inverse of a 2 x 2 Matrix** – Basic quick formula for finding the inverse of a simple square matrix
Linear Systems in a Matrix – Using a matrix to represent a linear system of equations
Elementary Row Operations – One method of solving systems of linear equations using matrices
Linear Systems (jump) – How to solve more than one linear equation at a time
Gaussian Elimination – Matrices – Using ERO to find the solution of a system of linear equations
Gauss-Jordan Elimination – Similar to Gaussian elimination but it goes a step further

Matrix Operations – List of ways of representing matrices
Identity Matrix – A matrix consisting of 1’s along its diagonal and zeros everywhere else
Matrix Addition – How to add two matrices together
Matrix Multiplication – How to multiply two matrices together
Properties of Matrix Operations – Properties of matrix addition, matrix multiplication and scalar multiplication
Scalar Multiplication – Multiplication of a matrix by a constant (scalar)

Sequences and Probability – Introductory page about sequences, series, and probability
Binomial Theorem – Expanding a binomial raised to an integer power
Pascal’s Triangle – A more visual way to find binomial coefficients
Counting Principle – Method of finding total number of ways a particular event can occur
Combinations – Number of ways a group of objects can be arranged irregardless of order
Permutations – Number of ways a group of objects can be arranged, with order being important
Factorial – A special type of function used to multiply integers together in a particular fashion
Probability – Terminology used throughout probability problems
Independent Events – An event that has no bearing on any previous or subsequent events
Union of Two Events – Also defined as the probability of events A or B occurring
Complementary Events – Probability of a complement is sort of the left over of the probability of an event
Sequences – An arrangement of numbers in a particular order based on a relationship between those numbers
Arithmetic Sequence – A sequence whose terms all have a common difference between consecutive terms
Geometric Sequence – A sequence whose consecutive terms have a common ratio
Summation Notation – A shortcut method of representing the sum of a large sequence of numbers
This workshop was designed to introduce teachers to teaching methods using the Algebra Brain.

**What were the best aspects of this program or activity?**

- Knowledgeable Instructors, Applicable in classroom
- Instruction. Hands on. Friendliness of presenters
- Usefulness of application
- Learning about and seeing and using BrainTrax. Getting to know what it is.
- Getting to see all the different aspects of the brain. The different lesson plans will be great.
- Hands on experience. Exposure to suing more technology as a teaching tool.
- Having sessions & lesson plans to work on specific topics.
- Explanations and examples.
- This is a great resource for myself as well as students to go in time of need.
- Having time to work on the Brain
- Getting ideas of lessons that could be used in the classroom

**What I will start doing in my classroom / school as a result of this program**

- Use this as a resource for my classes
- I hope to use this program at my school
- Apply [BrainTrax] as an instructional tool/resource
- Trying to have the students use BrainTrax as a learning tool.
- We are getting internet service just so our students can use this package. I will introduce my students to this so that they can use it as a tutorial.
- Use to supplement classroom work. Students can use at home as tutorial.
- I want to use this in my classroom and also train some of the others at my school.
- Hopefully, [take] algebra students to the lab to use as a supplement to lessons.
- Advertising this website to my students for them to access. Also I may use this as a tool of teaching or re-teaching a lesson.
- Use this as part of my instruction and have students use it for tutorials.
- Using the Brain as a tutorial and hopefully incorporate some lessons.
Commentary received via email:

Teachers:

Just thought I would let you know that we are up and running at the middle school - am introducing it this week to my students. It is great! Thanks.

Kenise Knight

Calculus Students

Almost everything is here... Even the stuff that the teacher didn't cover... FYI I'm a student in San Jose CA, going to DeAnza College. 8-) All hail google....

very easy to navigate, example problems go step-by-step, language used is easy to understand.

Many things! Very clever design--captures the essence of the material--good examples--"video" clips are cool--truly a tutorial which really helps

likes = I found a great example problem which will help me with the homework, but how to print it???
(I see no print button so I tried Print Screen to no avail) Sorry if my lack of computer skills are the problem.

it is very helpful to review things. especially for finals!

I like the easy to follow explanations of the topics.

Algebra Students

It provides a good review.

It has tons of useful information. The material presented was very comprehensive. Nothing seemed to be left out. The whole program is very worthwhile and helpful.

It was great I finally understand algebra

[I liked] the sound. It was helpful when they read off the problems.

Very helpful...liked the amount of information

It makes it sort of easy to understand

When you put your cursor on the black words it tells what it is
Overview

At the conclusion of the Math 4 class taught during the Fall Semester 2001, we issued a survey to all of the students. In this survey, we wanted to determine--from a student’s viewpoint--the effectiveness of the new BrainTrax Algebra Brain. Our determination of effectiveness hinges upon the premise that students who know more about algebra are going to be more successful in the course.

This survey was administered to 220 students. Of those, 92 students notified us that they did not use the system at all. These students were instructed not to complete the survey.

The following pages hold charts that reflect the responses of the remaining 128 students, both with and without regard of their visit frequency. Also in this report, we will draw conclusions based on student usage.

Each of the following pages contains the analysis of a single question, or premise, we submitted to the survey participants.
"I Learned Something New from the Algebra Brain."

In this question, we were seeking the student opinion about the teaching effectiveness of the Brain for concepts not presented by a human.

Discarding the "no opinion" votes as neutral, students agreed at **better than a 2 to 1 ratio** (59/23) that this system did in fact teach them "new" things about algebra. We were not surprised to observe that, in general, the more frequently the student visited the site, the more strongly they agreed with this statement. The following chart illustrates this point:

For those students with less than 5 visits, for whatever reason, a majority (18/12) disagreed with our premise that they learned something new. However, students falling into the 5 to 20 visit category **agreed at better than an 8 to 1 ratio** (33/4) that they did in fact learn something new at the site. An even stronger ratio (11/1) of agreement developed out of the 20 to 50 visit category.

Clearly the students who used this system did learn from it. While there are likely many reasons that that the students did not use the system in a uniformly large volume, for those who did, the learning experience with regard to algebra was enhanced.
“The Algebra Brain Clarified Something I Didn’t Understand in Class”

In this question, we wanted to know if our method of delivery was more effective than the instructor in clarifying a topic, once it was introduced by a human.

Figure 3a: The Algebra Brain Clarified Something Not Clear in Lecture

Discarding the “no opinion” votes as neutral, students agreed at **better than a 3 to 1 ratio** (65/17) that this system clarified concepts covered in class, but not fully understood by the student. The usage trend continues here, with students having a higher visit count trending toward stronger agreement of our premise.

A noticeable change in the pattern occurs here in the lowest volume category, where even students with a very few visits have a majority who agree with our premise (18/13). As the usage grows, so does the majority, with the 5 to 20 visit group **agreeing at better than an 8 to 1 ratio** (34/4), and the 20 to 50 group continuing with the strongest ratio (11/1).
"The Brain Interface Made It Easier to Find What I Wanted."

Next, we were interested in the effect of the Brain navigation tool itself. While we have observed students use this tool effectively, we were interested in seeing what the students thought of it in person.

Figure 4a: Brain Interface Helped

Again discarding the no-opinion votes, students reported at well over a 2 to 1 ratio (63/25) that this tool did make it easier for them to find the information they needed inside the site. The usage trend again continues here, with students having a higher visit count trending toward stronger agreement of our premise.

Figure 4b: Brain Interface Helped

The previously noted change in the pattern continues here; students with very few visits still maintain a majority who agree with our premise (21/13). As the usage grows to the next level, so does the majority, with the 5 to 20 visit group agreeing at nearly a 4 to 1 ratio (32/9), and the 20 to 50 visit group maintaining a strong majority opinion, at just under a 3 to 1 ratio (8/3).
"The Web Pages Were Easily Understood."

Regarding the content, we were interested in seeing how effective our writing and illustrations were in conveying the information to the student, so we asked about the understandability of the web pages. Notice that there is more to this question than meets the eye. Students attending UMR have a significant span in comprehension in both English and basic algebra. It is our intent that the web pages meet the needs of the entire range of student capabilities in these two areas.

![Figure 5a: Pages Easily Understood](image)

Again discarding the no-opinion votes, students reported at **better than a 4 to 1 ratio** (76/16) that our algebra content was effective in communicating at the student’s level of understanding.

![Figure 5b: Pages Easily Understood](image)

For this issue, even the infrequent visitors agreed strongly with our premise, giving a 4 to 1 ratio (28/7) in the agree column. As the usage grows, so does the majority, with the 5 to 20 visit group **agreeing at nearly a 5 to 1 ratio** (38/8), and the 20 to 50 visit group maintaining a strong majority opinion of agreement (9/1).
“The Web Pages Increased My Comprehension of Algebra.”

Next, we wanted to know how effective that communication was, so we asked the students (again), whether or not the web pages increased their comprehension of algebra. Note that we have asked this question three different ways to ensure that we are receiving valid responses about how effective this site is.

Staying with our pattern of discarding no-opinion votes, students this time reported at **better than a 4 to 1 ratio** (69/15) that our algebra content was effective in communicating at the student’s level of understanding.

For this issue, the strength of the infrequent visitors dropped sharply to just under a 2 to 1 ratio (19/11) in the agree column. However, in keeping with earlier observations, as the usage grows, so does the majority, with the 5 to 20 visit group **agreeing at over a 9 to 1 ratio** (38/4), and the 20 to 50 visit group joins the 50 to 100 visit group with no disagreement at all.
"I Would Like to Have More of My Classes Use Brain Technology to Support Them."

Finally, we asked students whether or not they wanted BrainTrax systems to support other classes. Student opinions were not quite as strong on this issue.

*Figure 7a: BrainTrax for Other Classes?*

By continuing to disregard the no-opinion votes, students reported as **nearly a 7 to 1 ratio** (68/10) that this type of system works for them, and they would like to see more of it in place.

In other words, we interpret this to mean that 68 out of the 220 students who were enrolled in Math 4, or **over 30% of the entire class of students, including those who didn't use it**, see value in this system for helping them to learn. This number changes to an **even more compelling 53%** when considering only the students who used the system.

Following the earlier demonstrated trend, those who use the system more tend to see it as more valuable. They may use it more *because* they see it to be more valuable.

*Figure 7b: BrainTrax for Other Classes?*

There was a large "no-opinion" group for this premise, giving rise to several questions that will require further study.
Individual BrainTrax Attributes

We were also interested in determining which components of the BrainTrax Algebra system the students found useful, so we asked them to evaluate each of the primary components with a rating between 1 and 5, with 1 being "not useful at all" and 5 being "most useful".

Figure 8: Usefulness of Brain Components

An observation about this chart is in order: Note that the Interactive Example and Testing System (IETS) is much more highly valued by the students than anything else on the chart.

As we had expected, the "terms" capability, where a term's definition pops up when the student places the mouse pointer over it, was also a popular item.

CONCLUSION

It is my somewhat biased opinion that these figures indicate that the BrainTrax system does, in fact, perform in the manner we intended it to. It can be enhanced to perform better. These enhancements will be addressed as they are identified, provided continued funding for this effort is available.

Comments regarding this report, including the conclusions drawn, are welcome.
The University of Missouri – Rolla's (UMR's) BrainTrax Algebra system is an online learning tool aimed at helping struggling students comprehend algebra. You can find it at:

http://braintrax.umr.edu

Click on "Algebra" at the top of the page to enter the online BrainTrax Algebra Brain. The Algebra Brain is a powerful tool designed to associate different concepts with each other in a visual web of knowledge. We have enhanced the visual web with substantial content. It is not a textbook. It is a way of learning mathematics.

What you hold in your hands is the entire body of knowledge contained in the Algebra Brain. Over 1000 pages of pure algebraic goodness, distilled into printer-friendly format. You might be wondering, "If it is available online, why print it out?"

First and foremost, in an effort to be more accessible to teachers and administrators, we have created this document so that everyone has a chance to see what the Algebra Brain has to offer. Oftentimes a computer terminal is not handy, or the right software has not been installed, or for some reason the hardware just does not work. In that case, we have BrainTrax Algebra in hand to provide instructors and administrators a glimpse of the many features offered in the Algebra Brain.

As mentioned earlier, the online Algebra Brain contains over 1000 pages of algebra. This is very difficult to comprehend in a website since you only see a small portion of the site at any given time. This monstrously thick volume hammers home the idea of 1000 pages of algebra, each page better than the last. Unfortunately, BrainTrax Algebra lacks one important component: it is not interactive like a website. You cannot point and click your way through this book.

Instructors can use BrainTrax Algebra as a reference to find information that might be of interest to their students (or themselves—there are a lot of interesting facts in BrainTrax Algebra not found in any algebra textbook). Having reviewed a given concept, the instructor can then point students to it in the Algebra Brain. Each concept in BrainTrax Algebra has a "thought-path" associated with it to guide users to that concept in the Algebra Brain (concepts in the Algebra Brain are called "thoughts").

We would like to stress that fact that BrainTrax Algebra is not a textbook! It does not contain any problem sets or exercises, it does not contain information on how to use a graphing calculator (found in many current textbooks), and it does not operate in a linear fashion like a typical algebra textbook. BrainTrax Algebra does have lengthy discussions on most topics covered in algebra. It has hundreds, if not thousands, of examples each with a detailed explanation of the solution. It has three levels of explanation for most concepts, suitable for the beginning algebra student through the more advanced students. In addition, it tries to connect algebra with the real world by illustrating how history has played an important role in the development of mathematics.

Although we have tried to duplicate the many features of the Algebra Brain in this document, some features are only available online. These include the audio commentary for the Basic-level instruction, the ever-shifting matrix of the Brain itself, and the instantaneous transfer between levels of instruction.

The BrainTrax website by itself has many features available to students and instructors alike. For students, we have sample exams, which are actual tests administered at UMR over the years. Each test also has a detailed solution provided. Students can work the tests, and then see how they did. Instructors can access a large number of documents, including all of the information here in BrainTrax Algebra, all in printer-friendly format.
We feel that we have created a highly usable resource for instructors and students. Each concept is covered in-depth in language the student can understand, while training the student to think rigorously about mathematics. It is never enough to simply fill-in-the-blanks of a formula; we encourage the student to understand why he or she obtained the given answer, a critical component in problem solving.

The BrainTrax Development Team has worked long and hard under the auspices of the University of Missouri – Rolla to provide superior mathematics instruction for the students of UMR. However, we feel it is in the best interests of UMR in particular and the rest of the state of Missouri in general to provide superior mathematical instruction to all students preparing to go on to college. If you would like more information about BrainTrax Algebra, the BrainTrax Development Team, or the Algebra Brain, please do not hesitate to contact us:

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Three levels of instruction – In an effort to reach as wide an audience as possible, we include three levels of instruction: Basic, Intermediate and Advanced. Sometimes a student needs a little extra explanation, therefore we provide the student with the following:

**Basic:** Suitable for students of Algebra I and up, Basic level instruction is just that, a rudimentary explanation of the concept at its most fundamental level. We still assume that the user understands basic arithmetic, however, along with some fundamental algebraic concepts such as variables and expressions. Basic level instruction also serves as remediation for higher-level students who require a little more explanation on a given concept. Some concepts are not even covered at the Basic level in a normal algebra course, but whenever possible, we still try to include a brief, non-technical description of the concept as well as showing the student why the given concept is important in later mathematics courses.

**Intermediate:** This level is most suitable for students of Algebra II and up. Advanced users who are struggling to understand the Advanced level of a concept may remediate to this level for a simpler, more detailed explanation of the given concept. Basic users who wish to increase their knowledge of a given concept may decide to learn at this level too, thus becoming even more proficient in algebra. Almost all concepts are covered at the Intermediate level; often with much more detail than at the Basic or even Advanced levels.

**Advanced:** The highest level of instruction in the Algebra Brain. The Advanced users are typically freshman at UMR who require some additional instruction outside the classroom in order to fully understand a given concept. Other users include students in high school who are taking a collegiate-level algebra course. Users who are struggling with mathematical comprehension can remediate themselves down to the Intermediate level, if they so choose. Every concept in the Algebra Brain is covered at the Advanced level. Some concepts—in particular formula sheets—are covered exclusively at the Advanced level simply because no additional explanation is required for Basic and Intermediate users.

**Unit Organization** – The information is structured into six units, roughly analogous to the algebra syllabus used on the UMR campus.

**Unit 1:** Fundamentals of Algebra – This unit covers pre-algebra concepts such as real numbers, absolute value, the Cartesian plane, algebraic expressions, and properties of arithmetic.

**Unit 2:** Linear Equations – Linear equations is the study of lines and slope. We introduce those concepts, along with linear inequalities and linear systems.

**Unit 3:** Functions – Central to the study of algebra is the concept of a function. We cover numerous types of functions, including rational functions, polynomial functions, transcendental functions, and inverse functions.

**Unit 4:** Quadratics – Although quadratics should technically fall under the category of functions, we assign the study of quadratic its own unit. In this unit we describe the various ways of solving a general quadratic equation, concluding with the Quadratic Formula. We also introduce the conic sections, a subset of quadratic equations. The conic sections include the circle, ellipse, parabola, and hyperbola.

**Unit 5:** Matrices and Determinants – Matrices are often used in algebra to represent systems of linear equations. We demonstrate how to use a matrix to solve a system of linear equations. A determinant is a real number associated with a square matrix (and only a square matrix!). Using determinants, we can find the area of a triangle, solve a system of linear equations (via Cramer’s Rule), and determine if three points are all on the same line.

**Unit 6:** Sequences and Probability – Sequences are special functions in algebra. We cover arithmetic and geometric sequences, sums of sequences, and summation notation. Probability is the determination of the likelihood of an event given some constraints. We discuss the Binomial Theorem, permutations, combination, factorials, complementary events and independent events.

Each unit is further subdivided into numerous concepts arranged in a sequential manner, although they do not appear sequentially in the Algebra Brain.
Concept Features:

Each concept in BrainTrax Algebra includes the following features.

Color coding – We have color coded BrainTrax Algebra to match the color codes we use in the online Algebra Brain. Each concept has a title and a level assigned to it at the top of the page. For instance, "Descartes's Rule of Signs" is green on the Intermediate level pages, and "Intermediate" (also in green) appears next to the title. The three colors used in the brain are:

Basic – purple
Intermediate – green
Advanced – blue

Goal / Terms / MathTrax – On each page containing detailed information about a given concept, we have reproduced the information found in the online Algebra Brain in a colored box at the top of the concept.

Goal – the learning objective for the concept

Terms – a list of words and phrases, along with the definitions, the student is expected to learn in the given concept

MathTrax – to add some historical perspective to the study of algebra, we include an interesting factoid about the development of mathematics over the course of several millennia

To find the number of real (and imaginary) roots of a polynomial function.

Descartes's Rule of Signs – a rule determining upper bounds to the number of positive zeros and to the number of negative zeros of a polynomial function.

positive real zeros – positive real $x$-values for which the function equals zero.

negative real zeros – negative real $x$-values for which the function equals zero.

variation in sign – two consecutive coefficients in a polynomial function having opposite signs.

In 1649, René Descartes (1590-1650) traveled to Sweden to tutor 20-year old Queen Christina. She preferred to start her days at 5 am. The combination of extremely cold winters and early rising resulted in Descartes contracting a fatal case of pneumonia. He died early spring of 1650. In 1666, his remains were exhumed and returned to France. The French ambassador received permission to remove Descartes's right forefinger from the corpse. Descartes's skull is said to have been removed by a guard and sold several times. Supposedly, Descartes's skull is currently on display at the Museé de l'Homme in the Palais de Chaillot in France.

Thought Path – After the Goal / Terms/ MathTrax box, we include a thought path. This is the route the student would travel to find the page in the Algebra Brain. Following this path in the Algebra Brain will reinforce the relationships between algebraic concepts. In addition, the thoughts in the Algebra Brain contain more features than are found here, simply because the online experience offers much more in terms of interactivity. Basic users, for instance, can have the text read to them, along with descriptions of the graphics to help students understand what they see.
To get to "Descartes's Rule of Signs" in the Algebra Brain, the user clicks on "Functions" followed by "Polynomial Functions" followed by "Real Zeros" and finally on "Descartes's Rule of Signs".

**Conceptual information** – Following the thought path, the user enters the realm of conceptual information. Here we explain the concepts in detail, often with fully-worked proofs. We also provide the relevant formulas the student needs to understand the material. Much of the conceptual information is given in plain (often very plain) English with little mathematical terminology. Often the most obscure mathematical formulas are better understood if redefined in terms the user can understand. We also include historical information to help the student realize that mathematics is continually being refined over the centuries.

However, we can use Descartes's Rule of Signs to find not only which roots are real and which roots are imaginary, but also the signs of those roots. Many problems in the real world only concern themselves with positive solutions. A good example is any projectile example involving time. Time is always a positive quantity (time moves forward, not backward). Thus, if we have a polynomial equation involving time that gives us negative roots, we can automatically disregard them because they do not make sense in the context of a time-based problem.

We must note the qualifier that the roots given by Descartes's Rule of Signs are all real. According to the Fundamental Theorem of Algebra, the degree of the polynomial tells us the total number of roots. Therefore, if Descartes's Rule of Signs yields us three real roots (positive, negative, or both) for a 5th degree polynomial, then we must conclude that the remaining 2 roots are imaginary.

**Formulas / Definitions** – Many ideas in algebra can be summed up in a simple formula or phrase. For instance, the Pythagorean Theorem is written as, "Given a right triangle with sides of lengths $a$, $b$, and $c$, where the side of length $c$ is opposite the right angle, the square of $c$ is the sum of the squares of $a$ and $b$, or in other words, $a^2 + b^2 = c^2$." Students are expected to not only memorize the formula, but also understand why it works, although sometimes that is well beyond the scope of algebra.

**Descartes's Rule of Signs**

Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$ be a polynomial with real coefficients and $a_0 \neq 0$.

1. The number of **positive real zeros** of $f$ is either equal to the number of variations in signs of $f(x)$ or less than that number by an even integer.

   This means that for every change of sign (from $+a_n$ to $-a_{n-1}$ to $+a_{n-2}$, etc), you will have a zero for each change in sign or two. If you do not have a zero for each change in sign, then you will have two less than the maximum number of zeros. If you do not have that many, then you will have four less than the maximum, etc... These are all for positive real zeros.

2. The number of **negative real zeros** of $f$ is either equal to number of variations of sign of $f(-x)$ or less than that number by an even integer.

   The same rule that applies to the positive real zeros applies to the negative zeros as well.
Notes – Many formulas and definitions are followed by notes. These are additional pieces of information the student needs to keep in mind when applying the formula or definition.

Notes: Variation in sign means that two consecutive coefficients have opposite signs.

Where there is only one variation in sign, Descartes’s Rule of Signs guarantees the existence of exactly one positive (or negative) real root.

Descartes’s Rule of Signs does not tell us what the zeros are. It simply states how many zeros we will find, as well as whether they will be positive or negative. We need to use other methods to actually find the zeros.

Examples – Most pages contain at least one example problem after a formula or definition is given. This is so the student can see for him or herself how the formula or definition is applied. Each and every example problem is accompanied by a step-by-step explanation of how the problem is solved. Many example problems are real-world situations, sometimes with disastrous consequences if the problem is solved incorrectly! Besides simply showing the student how to work a problem, we also strive to teach the student how to answer the question that is asked. Oftentimes, the mathematics is only a small part of the problem. If we obtain two answers, one positive and one negative, which one do we use?

Example

Use Descartes’s Rule of Signs to determine how many real solutions the following function has as well as if they are positive or negative.

\[ x^3 - 2x^2 - 5 = 0 \] given equation

solution

Our given equation has coefficients that are positive, negative, and negative again. This translates to only 1 variation in sign.

Descartes’s Rule of Signs tells us that the number of positive real solutions is equal to the number of changes in sign or less than that by an even number. Thus, we can have either 1 positive real root or 0 positive real roots (we can’t have less than 0 roots. That just wouldn’t make sense).

In order to determine our negative real roots, we have to replace “x” with “–x” in the function and simplify, noting any variations in sign again:

\[ (-x)^3 - 2(-x)^2 - 5 = -x^3 - 2x - 5 \] replace “x” with “–x”

All the coefficients are negative. Thus, we conclude from Descartes’s Rule of Signs that there are no negative real roots, since there are no variations in sign.

Thus we have at most, 1 positive real root. answer

Graphs / Pictures – Each graph and diagram in the Algebra Brain has been hand-crafted for maximum effect. Pictures really can be worth a thousand words if designed efficiently. We have strived to provide clear, concise images that relay as much information about a given situation as possible. Most diagrams include color to enhance the effect. For instance,

\[ 4x^6 - 3x^5 + 2x^4 - x^3 + 3x^2 + 4x + 7 \]

indicates how we determine the number of changes of sign in a polynomial expression so that we can then apply Descartes’s Rule of Signs.
Connections – Finally, we try to enhance the learning experience by providing connections between concepts. Many concepts have little or no meaning to the student now, but may become more and more important if the student wishes to learn higher mathematics. For instance, exponential functions may not seem to have any importance to a student learning algebra in 10th grade, but that same student will be very glad he or she learned about exponential functions in 10th grade when he or she studies calculus. Since the Algebra Brain was originally designed for algebra students on the UMR campus, we feel it is only fitting that we inform students that the concepts they learn today will be of great benefit to them when they study calculus later in their academic careers.

Property 4 is somewhat misleading. When you study calculus, you will find that it is appropriate to say that \((a / 0)\) "approaches" infinity. To see why, consider dividing \(a\) by a really small decimal number that is close to zero, such as 0.00001. Then you have:

\[
\frac{a}{0.00001} = \frac{a}{10^{-5}} \quad \text{rewrite denominator in exponential form}
\]

\[
= a(10^5) \quad \text{property of exponents}
\]

\[
= a(100,000) \quad \text{rewrite in standard notation}
\]

Dividing \(a\) by a really small decimal number is the same as multiplying \(a\) by a really large number. Thus, as the denominator gets closer and closer to zero, the evaluation of the expression becomes larger and larger, eventually "reaching" infinity when the denominator equals zero.

Features found only in the Algebra Brain:

All of the features described above can be found in the online Algebra Brain—located at braintrax.umr.edu—but the following features are found only in the Algebra Brain, since they are unique to the web-based learning environment and cannot be represented in a paper-based model.

The Brain – The online Algebra Brain contains a navigation feature known simply as "The Brain". This navigation tool allows students to quickly find the algebra concept they seek. Students need no more than a dozen clicks to go from any point in the Algebra Brain to any other point in the Algebra Brain.

Interactive Example and Testing System (IETS) – Authorized students can access our database of interactive example problems for each concept. Instructors who contact us can request userids and passwords for their students. Students are able to test their knowledge of the various concepts and applications, obtaining instantaneous feedback, as well as detailed explanations when they fail to solve a problem correctly.

Transfer between levels – Each concept in the Algebra Brain contains a menu with three buttons indicating "Basic", "Intermediate", or "Advanced" levels of knowledge. Students can click on one of the three buttons to be instantly transferred to the level of instruction they selected. Furthermore, from the BrainTrax homepage, they can choose a default level of instruction.

Audio – Basic level pages in the Algebra Brain contain an audio icon that—when clicked—reads the information on the page to the student. Graphs and diagrams are explained to the student so they understand what the graph or diagram represents.

"/" and "e" buttons – The "/" button on a page indicates that if a student clicks on the button, he or she will be transferred to another page on the same concept with more information about the concept. The "e" button is placed next to an example on a page and will transfer the student to another example of the same concept, though at a different instruction level.

Mouseover definitions – Bolded terms on a web page in the Algebra Brain can be moused over to reveal the definition of the word or phrase. This provides the student with instantaneous information about the term he or she is unfamiliar with instead of having to refer to a glossary. In addition, all terms are located at the top of a given web page for easy access.
Access 24/7 – The Algebra Brain is available to all Missouri K–12 students 24-hours a day, 7 days a week. To access the online Algebra Brain, simply enter "braintrax.umr.edu" (NO "www") into the address window of Internet Explorer 5.0+. Due to complex technical issues, the Algebra Brain can only be viewed on a PC-compatible machine running Internet Explorer 5.0 or higher. Once on our homepage, click on "Algebra" at the top of the page to enter the Algebra Brain.
Scavenger Hunt

Access BrainTrax by typing braintrax.umr.edu
Select ALGEBRA Brain.

1. What six main course topics are covered by the Algebra Brain?
   __________________  _________________  _________________
   __________________  _________________  _________________

2. Access FUNDAMENTALS OF ALGEBRA. What options can you choose from this thought path?
   __________________  _________________  _________________
   __________________  _________________

3. Select REAL NUMBERS and then select ABSOLUTE VALUE. Describe what happens when you click on the i-button.
   _____________________________________________________

4. Are you still in the thought ABSOLUTE VALUE? __________________

5. What is the difference between the thought page you first saw when you selected ABSOLUTE VALUE and the thought page you are on now?
   _____________________________________________________

6. List two ways you could tell the difference between these two pages:
   __________________  ________________________

7. List the GOAL for the ABSOLUTE VALUE page. __________________
   _____________________________________________________

8. What happens when you click the speaker icon on the left side of the page?
   _____________________________________________________

9. Navigate your way back to the COURSE CONTENT. Describe what you did to get there?
   _____________________________________________________

For the following questions, navigate to: FUNCTIONS → POLYNOMIAL FUNCTIONS → POLYNOMIAL OPERATIONS → POLYNOMIAL DIVISION.

    _____________________________________________________

11. Click on the Descending Grade button (next to the Basic button) and complete the activity. Describe what happened.
    _____________________________________________________
12. In the white box at the lower right corner of the "Brain" (the blue part at the top of the page), type the words **quadratic equations**. What happened?

____________________________________________________

13. Click on the EXAMPLE button. What is it about?

____________________________________________________

14. Close the EXAMPLE by clicking on the upper right close button. Click on the icon on the left side of the page. Describe what happens.

____________________________________________________

15. Click on the TERMS button at the top left of the page. List the new terms that are defined here.

____________________________________________________

16. Click MATHTRAX button at the top of the page. What is the purpose of this button?

____________________________________________________

17. Complete this activity twice. Describe any differences between the first time you completed the activity and the second time you completed the activity.

____________________________________________________
To multiply two or more polynomials together.

No definitions on this page.

Claudius Ptolemy (c.70-c.130 A.D.) was a famous Greek astronomer, geometer, and geographer. Ptolemy's great work, the *Almagest*, provides a comprehensive overview of Greek geometry and trigonometry. Ptolemy's geocentric (Earth-centered) view of the universe was so compelling that it remained in force for over 1400 years, until Copernicus set forth his heliocentric (sun-centered) view of the solar system. Ptolemy the geographer attempted to map the Known World at that time, but since his knowledge of geography was effectively limited to the Roman Empire, his maps were very inaccurate. He did, however, introduce the first systematic use of longitudes and latitudes. His directions for creating maps were still being used by cartographers as late as the Renaissance.

Just as we can use the area of a rectangle to help establish the multiplication of two numbers, we can also use the area of a rectangle (or more accurately, several rectangles) to help define multiplication of polynomials. Say that we wish to multiply the expression \(a + b + c + d\) by the expression \(x + y + z\), where \(a, b, c, d, x, y,\) and \(z\) represent variables, real numbers, or algebraic expressions.

Let's draw a rectangle with one side being of length \(a + b + c + d\) and the other side of length \(x + y + z\), as shown below:

```
  a   b   c   d
 x
 | ax | bx | cx | dx |
 y
 | ay | by | cy | dy |
 z
 | az | bz | cz | dz |
```

The total area of the largest rectangle is equal to the sum of the areas of all twelve of the smaller rectangles:

\[
\text{Total area} = ax + ay + az + bx + by + bz + cx + cy + cz + dx + dy + dz
\]

But, as with any rectangle, the total area is also equal to the product of its length and width:

\[
\text{Total area} = (a + b + c + d)(x + y + z)
\]

These two statements are equivalent to each other. We can use the Distributive Property to justify why this is true. If we distribute \((a + b + c + d)\) over \((x + y + z)\), we obtain:
\[(a + b + c + d)(x + y + z) = a(x + y + z) + b(x + y + z) + c(x + y + z) + d(x + y + z)\]

\[= ax + ay + az + bx + by + bz + cx + cy + cz + dx + dy + dz\]

which is the same as the first total area we arrived at above. This is what is known as the Extended Distributive Property.

**Extended Distributive Property**

To multiply two polynomials multiply each term in the first polynomial by each term in the second polynomial.

If one polynomial has \(n\) terms and another polynomial has \(m\) terms, then their product will have \(m \cdot n\) terms. From our rectangular example above, we can see that there are 4 terms multiplied by 3 terms, so our result has \(4 \cdot 3 = 12\) terms. If some of the terms are like terms, then we can combine them into a single term. For example, if our result gives us \(2x^2 + 3x^2\), then we can combine this into:

\[2x^2 + 3x^2 = (2 + 3)x^2 = 5x^2\]

After we simplify, we can end up with fewer terms than one or both factors, especially if we can combine like terms (which is one reason why we multiply polynomials together!).

Let's actually do a geometric example of this using some real numbers and a few variables thrown in for good measure.

**Example**

Find the area of the largest rectangle pictured below and simplify the result.

```
2u

3v

u   v   4
```

**solution**

First, let's go by the model at the top of the page and write the product in terms of the length \((u + v + 4)\) times the width \((2u + 3v)\) to get:
Area \quad = \quad (u + v + 4)(2u + 3v) \\
\quad = \quad (u)(2u) + (u)(3v) + (v)(2u) + (v)(3v) + (4)(2u) + (4)(3v) \\
\quad \text{apply Extended Distributive Property} \\
\quad = \quad 2u^2 + 3uv + 2uv + 3v^2 + 8u + 12v \\
\quad \text{simplify} \\
\quad = \quad 2u^2 + 5uv + 3v^2 + 8u + 12v \\
\quad \text{simplify by adding like terms} \\
\text{answer}

\text{Note:} \quad \text{While this problem involves only positive numbers (assuming that } u \text{ and } v \text{ are both positive), the Extended Distributive Property applies equally to polynomials involving negative terms. However, we must be extremely careful about where the negative signs go, as they sometimes have a tendency to get "lost" during the process of calculation.}

\text{Example}

Multiply \((3x - y + 4)(x + 11y - 7)\).

\text{solution}

This time, we have some negative signs, so we have to be a bit more cautious about how we approach this.

Each term of \(x + 11y - 7\) must be multiplied by \(3x\), \(-y\), and \(4\). We will have nine terms at first (because each polynomial has 3 terms and \(3 \cdot 3 = 9\)).

\[(3x - y + 4)(x + 11y - 7) = (3x)(x) + (3x)(11y) + (3x)(-7) + (-y)(x) + (-y)(11y) + (-y)(-7) + (4)(x) + (4)(11y) + (4)(-7)\]

\[= 3x^2 + 33xy - 21x - xy - 11y^2 + 7y + 4x + 44y - 28\]

\[= 3x^2 + 32xy - 11y^2 - 17x + 51y - 28\]

\text{answer}

\text{simplify by combining like terms}
Multiplying Polynomials  Intermediate

Goal

To multiply two or more polynomials together.

Terms

No definitions on this page.

MathTrax

Claudius Ptolemy (c.70-c.130 A.D.) was a famous Greek astronomer, geometer, and geographer. Ptolemy's great work, the *Almagest*, provides a comprehensive overview of Greek geometry and trigonometry. Ptolemy's geocentric (Earth-centered) view of the universe was so compelling that it remained in force for over 1400 years, until Copernicus set forth his heliocentric (sun-centered) view of the solar system. Ptolemy the geographer attempted to map the Known World at that time, but since his knowledge of geography was effectively limited to the Roman Empire, his maps were very inaccurate. He did, however, introduce the first systematic use of longitudes and latitudes. His directions for creating maps were still being used by cartographers as late as the Renaissance.

Functions → Polynomial Functions → Polynomial Operations → Multiplying Polynomials

Polynomials have a wide range of applications in geometry, particularly involving problems of area and volume. Most often, they crop up in problems with rectangular shaped surfaces and objects.

Consider a row of town houses on a downtown city street. Each house has the same height given by the expression $f + s$ (height of first story + height of second story). The widths of the three houses are given by $x$, $y$, and $z$, respectively. Thus the total area of the front of the town houses is given by $(f + s)$ multiplied by their widths, given by $(x + y + z)$.

One way to think of this total area is as the sum of the areas of six smaller rectangles (three first floors plus three second floors). This gives us the total area as:

$$\text{Area} = fx + fy + fz + sx + sy + sz \quad \text{total area of the front of the town houses}$$

Another way we can picture the total area is as the area of one large rectangle with base $x + y + z$ and height $f + s$. Then we use the usual geometric formula to find the area of a rectangle, $A = bh$, and get:

$$\text{Area} = (x + y + z)(f + s) \quad \text{total area of the front of the town houses}$$

We want to expand this expression, so we apply the Distributive Property, considering the factor $(x + y + z)$ as one unbreakable chunk (for now):

$$\text{Area} = (x + y + z)f + (x + y + z)s \quad \text{multiply (x + y + z) by each term in the second factor}$$
We apply the Distributive Property two more times to obtain:

\[
\text{Area} = fx + fy + fz + sx + sy + sz \quad \text{this is the same as the expression at the top of the page}
\]

What we have just demonstrated is how to multiply two polynomials together. This involves applying the Distributive Property multiple times. Since that is the case, we say that multiplying polynomials is an application of the Extended Distributive Property:

**Extended Distributive Property**

To multiply two polynomials multiply each term in the first polynomial by each term in the second polynomial.

If one polynomial has \( m \) terms and the second polynomial has \( n \) terms, then there will be \( m \cdot n \) terms in the product before we combine like terms.

This property works equally well if we have more than two polynomials. For example, if we wanted to multiply three binomials together, then we apply the Extended Distributive Property to the first two binomials, leaving the third binomial alone. What we then end up with is a binomial multiplied by a trinomial (the result of multiplying two binomials). So we then apply the Extended Distributive Property again. Our result will most likely be a polynomial with five terms (after we combine like terms).

Let's look at a practical example involving finding the maximum volume of a box.

**Example**

A piece of metal 24 inches by 30 inches is made into a box by cutting out squares of side \( x \) from each corner. Let \( V(x) \) be the volume of the box. Find a polynomial formula in standard form for \( V(x) \). Use a graph of \( V(x) \) to approximate the value of \( x \) that will give us the maximum volume.

![Diagram of a box with dimensions labeled: 30 inches by 24 inches by 2x inches]  

**solution**

We know from geometry that the volume of a box is given by the formula \( V = lwh \). When we fold up the sides of the box, the box will have a length of 30 – 2x inches, a width of 24 – 2x inches, and a height of \( x \) inches.
So in algebraic terms, we can write down the formula as:

\[
V(x) = (30 - 2x)(24 - 2x)x
\]

\[
= (30 - 2x)(24x - 2x^2)
\]

\[
= 720x - 60x^2 - 48x^2 + 4x^3
\quad \text{Extended Distributive Property}
\]

\[
= 4x^3 - 108x^2 + 720x
\quad \text{combine like terms}
\]

\[
\text{write in standard form}
\]

This gives us a formula for the volume of the box for any value of \(x\). Note that since we are talking about a three-dimensional object (a box), we have a third power of \(x\) in our formula.

In order to find the approximate value of \(x\) that will give us the maximum volume, we need to turn to a graphing utility such as MathCAD to help us:

The blue lines on the graph indicate the maximum volume and the value of \(x\) for which the function \(V(x)\) reaches the maximum. According to this graph, we can see that when \(x \approx 4.5\), the function \(V(x)\) reaches its maximum. \(\text{answer}\)

To find out what this maximum volume is, we plug in \(x = 4.5\) into our function and see what we get:

\[
V(4.5) = 4(4.5)^3 - 108(4.5)^2 + 720(4.5)
\quad \text{substitute in } x = 4.5 \text{ into our volume function } V(x)
\]

\[
= 4(91.125) - 108(20.25) + 720(4.5)
\quad \text{simplify}
\]

\[
= 1417.5
\quad \text{simplify using a calculator}
\]

Thus, when \(x = 4.5\), the volume of the box is 1,417.5 cubic inches.

\(\text{Note:}\) We could use calculus to find out exactly when the volume reaches its maximum value, but that is a little beyond the scope of the Algebra Brain.
To multiply two or more polynomials together.

No definitions on this page.

Claudius Ptolemy (c.70-c.130 A.D.) was a famous Greek astronomer, geometer, and geographer. Ptolemy's great work, the *Almagest*, provides a comprehensive overview of Greek geometry and trigonometry. Ptolemy's geocentric (Earth-centered) view of the universe was so compelling that it remained in force for over 1400 years, until Copernicus set forth his heliocentric (sun-centered) view of the solar system. Ptolemy the geographer attempted to map the Known World at that time, but since his knowledge of geography was effectively limited to the Roman Empire, his maps were very inaccurate. He did, however, introduce the first systematic use of longitudes and latitudes. His directions for creating maps were still being used by cartographers as late as the Renaissance.

The Distributive Property is used to multiply two polynomials together. We treat one of the polynomials as a single quantity and multiply that single quantity by both terms in the second polynomial. Essentially, we apply the Distributive Property *twice*. For example, if we want to multiply $(3x + 7)$ by $(4x - 1)$, then we have the following:

$$(3x + 7)(4x - 1) = 3x(4x - 1) + 7(4x - 1)$$

let $(4x - 1)$ be a single quantity and apply the Distributive Property

$$= (3x)(4x) - (3x)(1) + (7)(4x) - (7)(1)$$

apply the Distributive Property again

$$= 12x^2 - 3x + 28x - 7$$

multiply terms

$$= 12x^2 + 25x - 7$$

add like terms to simplify

If we take a closer look at what we did, we will see a pattern emerge. We multiplied the First terms of the polynomials together, followed by the Outside terms of the polynomials, followed by the Inside terms, and followed finally by the Last terms. We call this the FOIL method of multiplying two binomials together. Note that the outer (O) and inner (I) terms are like terms and can thus be combined into one term.

**FOIL Method**

First terms: $(3x)(4x)$

Outside terms: $(3x)(-1)$

Inside terms: $(7)(4x)$

Last terms: $(7)(-1)$

This method works great for simple binomials, but what happens when we try to multiply two *trinomials* together? It turns out that we perform a similar series of operations. In effect, we need to multiply each term of the first trinomial by each term of the second trinomial.
**Extended Distributive Property**

To multiply two polynomials multiply each term in the first polynomial by each term in the second polynomial.

**Example**

Multiply \((x^2 - 4x + 6)\) by \((5x^2 + 2x - 3)\).

**solution**

\[(x^2 - 4x + 6)(5x^2 + 2x - 3) = \frac{x^2(5x^2 + 2x - 3) - 4x(5x^2 + 2x - 3) + 6(5x^2 + 2x - 3)}{6(5x^2 + 2x - 3)}\]

\[= (5x^4 + 2x^3 - 3x^2) - (20x^3 + 8x^2 - 12x) + (30x^2 + 12x - 18)\]

\[= 5x^4 - 18x^3 + 19x^2 + 24x - 18\]

**Note:** When multiplying out polynomials, it is a good idea to not skip any steps. By taking care to multiply each and every term together, it is far less likely that you will make any sign errors or other mistakes.

We can easily (so to speak) extend the multiplication of trinomials to polynomials with even more terms. The trick is to be careful and take your time to avoid any inadvertent errors. The most common errors in multiplying any two polynomials are sign errors.

In general, if one polynomial has \(m\) terms and the other polynomial has \(n\) terms, then the total number of terms obtained by multiplying the two polynomials together will be \(m \cdot n\) terms. If we multiplied this result by a polynomial with \(q\) terms, then we would obtain a polynomial that contains \(m \cdot n \cdot q\) terms.