



## Algebra Brain Summary

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This is a list of all the thoughts located in the Algebra Brain. Each thought is followed by a statement indicating the content associated by that thought. Thoughts followed by (jump) are jump thoughts, which are located on the left hand side of the central thought in the Brain matrix. Jump thoughts often take the user to entirely different sections of the Brain or provide reference information about the central thought.

This compilation of thoughts only includes those thoughts listed under **Content** in the Algebra Brain.

**Algebra Glossary (jump)** – Several definitions of common terms used throughout Algebra  
**Algebra Alphabet** – A list of how each letter of the alphabet is used throughout Algebra

**Fundamentals of Algebra (jump)** – The basics needed to fully understand what Algebra is all about  
**Course Content (jump)** – The 5 basic concepts covered in all Algebra courses, from 8th grade to 13th grade  
**Algebraic Expressions** – The expression is one of the basic units used in Algebra, is part of an algebraic function  
**Basic Rules of Algebra** – Covers many of the fundamental properties exhibited by different operators in Algebra  
**Properties of Equality** – What equality really means  
**Properties of Exponents** – How to manipulate exponents to simplify algebraic expressions  
**Exponential Functions (jump)** – Basic definition of the exponential function  
**Properties of Fractions** – How to manipulate fractions to simplify algebraic expressions  
**Rational Functions (jump)** – One polynomial function divided by another  
**Properties of Inequalities** – A list of common properties that are obeyed by inequalities  
**Linear Inequalities (jump)** – How to solve an inequality instead of an equation  
**Properties of Zero** – Fundamentally, very, very important in order to fully understand Algebra  
**Cartesian Plane** – A brief review of the Cartesian Plane, including the Distance and Midpoint Formulas  
**Radicals** – An introduction to square roots, cube roots, and other roots  
**Properties of Radicals** – How to manipulate radicals to simplify algebraic expressions  
**Real Numbers** – Introduction to different number sets used throughout Algebra  
**Absolute Value** – Introduction to the absolute value operator and what it really means  
**Ordering Real Numbers** – How real numbers relate to each other with respect to the origin

**Functions** – Introduction to one of the fundamental concepts in Algebra  
**Function Terminology (jump)** – A list of common terms associated with functions  
**Inverse Functions** – Interchanging the domain and range of a function  
**Composition of Functions (jump)** – A function of a function  
**Finding the Inverse** – How to find the inverse of a function  
**Horizontal Line Test** – The test which determines if a function has an inverse  
**Polynomial Functions** – Functions involving a sum of powers of  $x$   
**Fundamental Theorem of Algebra (jump)** – The number of roots of a polynomial function equals the highest power of  $x$   
**Complex Numbers** – Numbers the complex plane, of which the real numbers are a subset  
**Polynomial Operations** – How to manipulate polynomials  
**Adding/Subtracting** – How to add and subtract two (or more) polynomials  
**Multiplying Polynomials** – How to multiply two (or more) polynomials  
**Special Product Patterns (jump)** – A list of commonly occurring products that occur in Algebra  
**Polynomial Division** – How to divide two polynomials  
Synthetic Division – Technique used to divide polynomials of a particular form  
Factor Theorem – A polynomial  $f(x)$  has a factor  $(x - k)$  if and only if  $f(k) = 0$   
Remainder Theorem – If a polynomial  $f(x)$  is divided by  $(x - k)$ , then the remainder is  $r = f(k)$   
**Real Zeros** – The real solutions of a polynomial equations (as opposed to the complex solutions)  
**Descartes's Rule of Signs** – How to determine the number of positive and negative roots of a polynomial equation  
**Rational Zero Test** – How to find possible roots of a polynomial function with integer coefficients  
**Special Factoring Patterns** – A list of commonly occurring factoring patterns that occur in Algebra  
**Special Product Patterns** – A list of commonly occurring products that occur in Algebra  
**Multiplying Polynomials (jump)** – How to multiply two (or more) polynomials  
**Rational Functions** – One polynomial function divided by another  
**Conic Sections (jump)** – A look at some practical examples of rational functions  
**Polynomial Division (jump)** – Intimately tied with rational functions  
**Properties of Fractions (jump)** – Necessary for understanding how rational functions work  
**Asymptotes** – What they are and how they are used to help sketch rational functions  
**Partial Fractions** – How to convert a rational function into a sum of two fractions  
**Distinct Linear Factors** – Denominator of rational function is composed of distinct linear factors  
**Distinct Quadratic Factors** – Denominator of rational function is composed of irreducible quadratic factors  
**Mixed Factors** – Denominator of rational function contains both linear and quadratic factors (distinct and/or repeating)  
**Repeated Linear Factors** – Linear factors that repeat themselves in the denominator of a rational function  
**Repeated Quadratic Factors** – Irreducible repeating quadratic factors in the denominator of a rational function  
**Sketching Rational Functions** – How to sketch the graph of a rational function

**Transcendental Functions** – A category of functions that includes both exponential and logarithmic functions

**Exponential Functions** – Basic definition of the exponential function

**Properties of Exponents (jump)** – The various rules governing the behavior of exponents

**Exponential / Logarithmic Equations** – How to use logarithms to solve mathematical equations

Logarithmic/Exponential Models – Applications of logarithmic/exponential functions

Exponential Growth and Decay – Very common application of logs

Gaussian – Model which produces the bell-shaped curve used in statistical analysis

Logarithmic – Used in a wide variety of applications including earthquakes, sound, and time of death

Logistics Growth – Model used to accurately represent population growth in an environment

**Logarithmic Functions** – The inverse of an exponential function

**Properties of Logarithms (jump)** – Various rules governing behavior of logarithmic functions

**Exponential / Logarithmic Equations** – How to use logarithms to solve mathematical equations

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**Natural Base  $e$**  – Definition of logarithm of base  $e$

**Translations and Combinations** – Moving graphs around the coordinate plane

**Arithmetic Combinations** – How to add, subtract, multiply, and divide two functions

**Composition of Functions** – A function of a function

**Reflections in Coordinate Axes** – How to reflect a function around either the vertical or horizontal axis

**Vertical and Horizontal Shifts** – How to move a function vertically and horizontally around a graph

**Quadratics** – Second-degree polynomial functions

**Conic Sections** – Classification of conic sections by the discriminant of a quadratic function

**Circles** – Definition of a circle and its related equation

**Ellipses** – Definition of ellipse and its related equation

**Hyperbolas** – Definition of hyperbola and its related equation

**Parabolas** – Definition of a parabola and its related equation

**Translations of Conics** – Moving the graph of a conic section around in the plane

**Circle Translation** – How to move a circle around in the plane

**Ellipse Translation** – How to move an ellipse around in the plane

**Hyperbola Translation** – How to move a hyperbola around in the plane

**Parabola Translation** – How to move a parabola around in the plane

**Quadratic Equations** – One of the most important applications of polynomial functions

**1. Factoring** – Factor the quadratic, then set each factor equal to zero to find the roots

**2. Extracting Square Roots** – Extracting a square root from a quadratic equation

**3. Completing the Square** – Transforming a quadratic into a perfect square

**4. Quadratic Formula** – The granddaddy of all methods of solving a quadratic when nothing else works

**Linear Equations** – Equations that involve variables raised to the first power only

**Linear Inequalities** – How to solve an inequality instead of an equation

**Absolute Value Inequalities** – Inequalities involving absolute value operator

**Polynomial Inequalities** – How to find test intervals for solving polynomial inequalities

**Rational Inequalities** – Solving inequalities that involve one polynomial divided by another

**Properties of Inequalities (jump)** – A list of common properties that are obeyed by inequalities

**Linear Systems** – How to solve more than one linear equation at a time, often involving two or more variables

**Linear Systems in a Matrix (jump)** – Relationship between linear equations and matrices

**Graphical Approach** – How to solve a linear system from a graph of the equations

**Graphical Interpretation of Systems (jump)** – What linear systems have to do with the real world

**Method of Elimination** – How to solve a linear system of equations by eliminating a variable

**Multivariable Linear Systems** – Transformation of a linear system into row-echelon form

**Substitution Method** – Summary of how to solve a system of linear equations using back-substitution

**Systems of Inequalities** – Solving and graphing systems of linear inequalities

- Lines and Slope** – Various formulas involving lines and their slopes
- Equations of Lines** – Several different ways of writing equations of lines
- Parallel and Perpendicular Lines** – Basic definitions
- Point-Slope Form of a Line** – One form of an equation representing a line
- Slope-Intercept Form of a Line** – One form of an equation representing a line

## **Matrices and Determinants** – Basic definition of a matrix

- Determinants** – A special operation performed on a matrix
- Properties of Determinants (jump)** – A list of properties that we can use to simplify the process of finding determinants
- Applications of Determinants** – Several useful ways to apply matrices
- Area of Triangle** – How to find the area of a triangle with the given vertices using determinants
- Cramer's Rule** – Solving a system of linear equations using determinants
- Lines in Plane** – How to tell if three points lie on the same line and a method for finding the equation of a line
- Triangular Matrix** – A handy form of matrix used to quickly solve linear systems of equations
- Inverse of a Square Matrix** – Analogous to the inverse of a function
- Finding Matrix Inverse** – How to find the inverse of a matrix
- Inverse of a 2 x 2 Matrix** – Basic quick formula for finding the inverse of a simple square matrix
- Linear Systems in a Matrix** – Using a matrix to represent a linear system of equations
- Elementary Row Operations** – One method of solving systems of linear equations using matrices
- Linear Systems (jump)** – How to solve more than one linear equation at a time
- Gaussian Elimination – Matrices** – Using ERO to find the solution of a system of linear equations
- Gauss-Jordan Elimination – Similar to Gaussian elimination but it goes a step further
- Matrix Operations** – List of ways of representing matrices
- Identity Matrix** – A matrix consisting of 1's along its diagonal and zeros everywhere else
- Matrix Addition** – How to add two matrices together
- Matrix Multiplication** – How to multiply two matrices together
- Properties of Matrix Operations** – Properties of matrix addition, matrix multiplication and scalar multiplication
- Scalar Multiplication** – Multiplication of a matrix by a constant (scalar)

## **Sequences and Probability** – Introductory page about sequences, series, and probability

- Binomial Theorem** – Expanding a binomial raised to an integer power
- Pascal's Triangle** – A more visual way to find binomial coefficients
- Counting Principle** – Method of finding total number of ways a particular event can occur
- Combinations** – Number of ways a group of objects can be arranged irregardless of order
- Permutations** – Number of ways a group of objects can be arranged, with order being important
- Factorial** – A special type of function used to multiply integers together in a particular fashion
- Probability** – Terminology used throughout probability problems
- Independent Events** – An event that has no bearing on any previous or subsequent events
- Union of Two Events** – Also defined as the probability of events A or B occurring
- Complementary Events** – Probability of a complement is sort of the left over of the probability of an event
- Sequences** – An arrangement of numbers in a particular order based on a relationship between those numbers
- Arithmetic Sequence** – A sequence whose terms all have a common difference between consecutive terms
- Geometric Sequence** – A sequence whose consecutive terms have a common ratio
- Summation Notation** – A shortcut method of representing the sum of a large sequence of numbers