Name Key

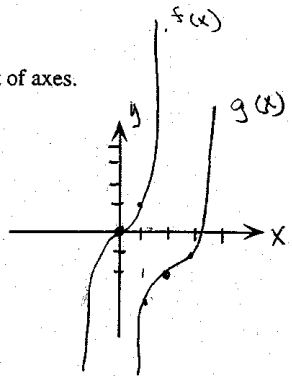
MATH 4
Exam 3
September 30, 1997

Instructor _____
Section _____

Show All work to receive partial/full credit.

[6] 1. Sketch the graphs of $f(x) = x^3$ and $g(x) = (x-2)^3 - 2$ on the same set of axes.

shift right 2 & down 2



[5] 2. Divide by long division. $(6x^3 + 10x^2 + x + 8) \div (2x^2 + 1)$

$$2x^2+1 \overline{)\,6x^3+10x^2+x+8} \qquad 3x+5 + \frac{-2x+3}{2x^2+1}$$

$-6x^3 \quad -3x$

$10x^2 - 2x + 8$

$-10x^2 \quad +5$

$3x+5+\dfrac{-2x+3}{2x^2+1}$

[5] 3. Use synthetic division to find $f(-2)$ for $f(x) = 4x^3 - 13x + 10$.

-2 | 4 0 −13 10
−8 16 −6
4 −8 3 ‖ 4

$4x^2 - 8x + 3$ R4

4. On parts a), b), and c), refer to the function $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$.

[4] a) Determine the left and right hand behavior of the graph of f.

falls to the left and rises to the right

[4] b) Use Descarte's Rule of Signs to determine the possible number of positive and negative real zeros of f.

$f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$ 3 or 1 positive

$f(-x) = -4x^5 - 8x^4 + 5x^3 + 10x^2 - x - 2$ 2 or 0 negative

[4] c) Use the Rational Zeros Test to list all the possible rational zeros of f.

p ±2, ±1

q ±4, ±2, ±1

Possible roots $\frac{1}{4}, -\frac{1}{4}, \frac{1}{2}, -\frac{1}{2}, 1, -1, 2, -2$

5. True or false. For the function $f(x) = 2x^3 - 3x^2 - 12x + 8$:

[3] a) $x = 4$ is an upper bound for the zeros of f.

true

[3] b) $x = -1$ is a lower bound for the zeros of f.

false

$4 \mid 2 \quad -3 \quad -12 \quad 8$
$\quad\quad 8 \quad 20 \quad 32$
$2 \quad 5 \quad 8 \quad \| 40$

$-1 \mid 2 \quad -3 \quad -12 \quad 8$
$\quad\quad -2 \quad 5 \quad 7$
$2 \quad -5 \quad -7 \quad 15$

[10] 6. Find ALL the zeros of the function $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$.

$-3 \mid 1 \quad 6 \quad 10 \quad 6 \quad 9$
$\quad\quad -3 \quad -9 \quad -3 \quad -9$
$1 \quad 3 \quad 1 \quad 3 \quad \| 0$

$(x+3)(x^3+3x^2+x+3) = (x+3)[x^2(x+3)+1(x+3)]$

$(x+3)(x+3)(x^2+1)$

Zeros are: $-3, -3, i, -i$

[5] 7. Find a polynomial function with integer coefficients that has 4 and $3i$ as zeros.

$(x-4)(x-3i)(x+3i) = (x-4)(x^2+9)$

$= x^3 - 4x^2 + 9x - 36$

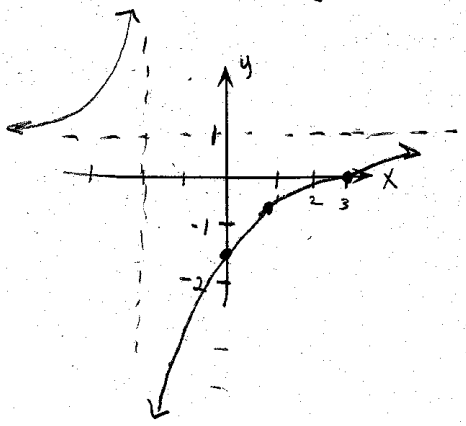
[12] 8. For the function $f(x) = \frac{x-3}{x+2}$ do the following:

a) Find the y-intercept;
b) Find the x-intercept;
c) Find the vertical asymptote(s);
d) Find the horizontal asymptote(s);
e) Plot additional points;
f) Graph the function.

a) $x = 0 \Rightarrow f(x) = -\frac{3}{2}$

b) $y = 0 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

c) $x + 2 = 0 \Rightarrow x = -2$



d) as $x \to \infty$ $f(x) \to 1$

e) $x = 1 \Rightarrow y = -\frac{2}{3}$

$x = -1 \quad y = -4$

$x = -3 \quad y = 6$

[5] 9. a) Set up the partial fraction decomposition for: $\frac{x^2+3}{(x-1)(x^2+1)}$ **DO NOT SOLVE**

$$\frac{x^2+3}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

[8] b) Write the partial fraction decomposition for $\frac{x+3}{(x^2+x-2)}$ **SOLVE FOR A, B, C, etc.**

$$\frac{x+3}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$x+3 = A(x-1) + B(x+2)$$

let $x = 1$: $4 = 3B \Rightarrow B = \frac{4}{3}$

let $x = -2$: $1 = -3A$, $-\frac{1}{3} = A$

$$\frac{x+3}{x^2+x-2} = -\frac{1}{3(x+2)} + \frac{4}{3(x-1)}$$

[10] 10. Find the center, and vertices of the hyperbola and sketch its graph.

$$9x^2 - y^2 - 36x - 6y + 18 = 0$$

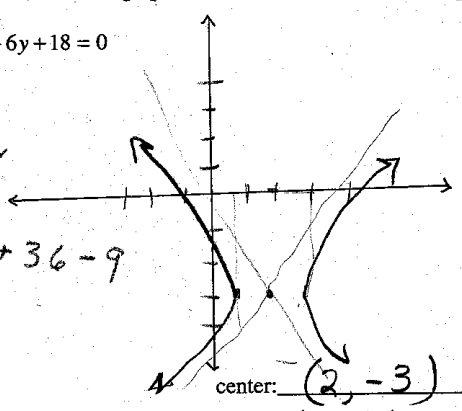
$9x^2 - 36x - y^2 - 6y = -18$

$9(x^2 - 4x\ \ ) - (y^2 + 6y\ ) = -18$

$9(x^2 - 4x + 4) - (y^2 + 6y + 9) = -18 + 36 - 9$

$\frac{9(x-2)^2}{9} - \frac{(y+3)^2}{9} = \frac{9}{9}$

$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$

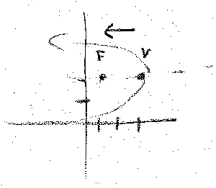


center: $(2, -3)$

vertices: $(1, -3)(3, -3)$

[8] 11. Find an equation of the parabola with vertex $(3, 2)$ (h,k) and focus $(1, 2)$.

$p = 1 - 3 = -2$



$(y-k)^2 = 4p(x-h)$

$(y-2)^2 = 4(-2)(x-3)$

$(y-2)^2 = -8(x-3)$

[8] 12. Identify the conic section by its equation. (2 points each)

A. $12x^2 + 20y^2 - 12x + 40y - 37 = 0$ ellipse

B. $4x - y^2 - 2y - 33 = 0$ parabola

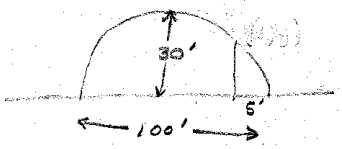
C. $9y^2 - x^2 + 2x + 54y + 62 = 0$ hyperbola

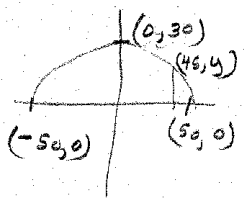
D. $3x^2 + 3y^2 - 5x + 4y = 0$ circle

BONUS 5 POINTS

A semielliptical arch over a tunnel for a road through a mountain has a major axis of 100 feet and a height at the center of 30 feet.

a) Make a sketch to solve the problem. Draw a rectangular coordinate system on the tunnel with the center of the road entering the tunnel at the origin. Identify the coordinates of the known points.





(b) Find an equation of the elliptical tunnel.

$$\frac{x^2}{50^2} + \frac{y^2}{30^2} = 1$$

(c) Determine the height of the arch 5 feet from the edge of the tunnel.

$$\frac{45^2}{50^2} + \frac{y^2}{30^2} = 1$$

$$(45^2)(30^2) + y^2(50^2) = 50^2(30^2)$$

$$y^2 = \frac{50^2(30^2) - 45^2(30)^2}{50^2}$$

$y^2 = 171$

$y \approx 13.08$ ft