Math 4
Exam 1
September 1, 1998

Instructor: Frank
Class Time: 8:30a

Name: [Name]

Show your work.

1. Solve for x

   (4) a) \( \frac{7}{2x + 1} - \frac{8x}{2x - 1} = 4 \)

   \[
   \frac{7(2x - 1) - 8x(2x + 1)}{(2x + 1)(2x - 1)} = 4
   \]

   \[
   14x - 7 - 16x^2 - 8x = 4(4x^2 - 1)
   \]

   \[
   6x = 7 + 4 = 11
   \]

   \[
   x = \frac{11}{6}
   \]

   (4) b) \( |x^2 + 6x| = 3x + 18 \)

   \[
   x^2 + 3x - 18 = 0
   \]

   \[
   \frac{x^2 + 3x + 18}{(x - 3)(x + 6)} = 0
   \]

   a) \( x = \frac{11}{6} \)

   b) \( x = \pm 3, -6 \)

2. Solve for \( f \) (Answer must be in simplest fractional form.)

   (3) \( \frac{q}{p} = \frac{f}{q - f} \)

   \[
   pq - pf = qf
   \]

   \[
   p \frac{q}{q - f} = qf
   \]

   \[
   \frac{pq}{q - f} = \frac{qf}{1}
   \]

   \[
   f = \frac{pq}{p + q}
   \]

3. Solve for \( w \) by factoring (Show work for credit.)

   (3) \( 9w^2 + 6w - 8 = 0 \)

   \[
   (3w - 2)(3w + 4) = 0
   \]

   \[
   w = \frac{2}{3}, -\frac{4}{3}
   \]

4. Solve by completing the square. (Show work.) Answer must be in simplest radical form or simplest \( a + bi \) form

   (3) \( 3x^2 - 8x + 2 = 0 \)

   \[
   x^2 - \frac{8}{3}x + \frac{16}{9} = -\frac{2}{3} + \frac{16}{9}
   \]

   \[
   \left( x - \frac{4}{3} \right)^2 = \frac{10}{9}
   \]

   \[
   x - \frac{4}{3} = \pm \frac{\sqrt{10}}{3}
   \]

   \[
   x = \frac{4 \pm \sqrt{10}}{3}
   \]
5. Solve by quadratic formula (Answer must be in simplest radical form, simplest $a+bi$ form, or simplest fractional form.)

\[ 5x^2 + x + 1 = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{-19}}{10} \]

\[ y = -1 \pm i \sqrt{19} \]

\[ x = \frac{-1 \pm i \sqrt{19}}{10} \]

6. Solve the following inequalities. Graph the solution and write your answer using interval notation.

(5) a. \[ 2x + 7 < 3 \]
\[ 2x < -4 \]
\[ x < -2 \]

Interval \((-\infty, -2)\)

(5) b. \[ -2 < 3x + 1 < 10 \]
\[ -3 < 3x < 9 \]
\[ -1 < x < 3 \]

Interval \((-1, 3)\)

(5) c. \[ \frac{|x-3|}{2} \geq 5 \]
\[ \frac{x-3}{2} \leq -5 \text{ or } \frac{x-3}{2} \geq 5 \]
\[ x-3 \leq -10 \text{ or } x-3 \geq 10 \]
\[ x \leq -7 \text{ or } x \geq 13 \]

Interval \((-\infty, -7] \cup [13, \infty)\)

(5) d. \[ \frac{3x-5}{x-5} \geq 4 \]
\[ \frac{3x-5}{x-5} - \frac{4(x-5)}{x-5} \geq 0 \]
\[ \frac{3x-5-4x+20}{x-5} \geq 0 \]
\[ \frac{-x+15}{x-5} \geq 0 \]

Critical numbers: \[ x = 5, x = 15 \]
7. Perform the operation and write the result in standard \((a + bi)\) form

\[
\frac{(1+i)}{i} - \frac{3}{4-i} = \frac{(1+i)(4-i) - 3i}{4i - i^2} = \frac{4 + 3i - i^2 - 3i}{4i - i^2} = \frac{4 + 2i + 1}{4i + 1} = \frac{5}{4i + 1}, \quad \frac{-4i + 1}{4i + 1}
\]

\[
= \frac{-20i + 5}{16 + 1} = \frac{-20i + 5}{17} \quad \text{Std. Form} \quad \frac{5}{12} - \frac{20}{17}i
\]

8. Find all solutions of the equation.

\[
6\left(\frac{s}{s+1}\right)^2 + 9\left(\frac{s}{s+1}\right) - 6 = 0
\]

\[
\frac{S_1}{s+1} = \frac{-5 \pm \sqrt{25 + 144}}{12} = \frac{-5 \pm 13}{12} = \frac{-2}{3} \text{ or } \frac{-8}{3}
\]

\[
\frac{S_2}{s+1} = \frac{3}{8} \quad \frac{s}{s+1} = -\frac{3}{2}
\]

\[
S_1 = 2 + 2 \quad S_2 = -3s - 3
\]

\[
S = \frac{-9}{5}
\]

9. Find all solutions of the equation.

\[
\sqrt{x+1} - 3x = 1
\]

\[
x + 1 = (1 + 3x)^2 - 1 + 6x + 9x^2
\]

\[
9x^2 + 5x = 0
\]

\[
x (9x + 5) = 0
\]

\[
x = 0 \quad \text{or} \quad x = -\frac{5}{9}
\]

\[
x = 0
\]

10. On the first part of a 350-kilometer trip, a salesperson traveled 2 hours and 15 minutes at an average speed of 100 kilometers per hour. Find the average speed required for the remainder of the trip if the salesperson needs to arrive at the destination in another hour and 20 minutes.

\[
\begin{align*}
\text{2.25 km/h} \\
100 \text{ km} \\
350 \text{ km}
\end{align*}
\]

\[
2.25 \text{ hrs} \times 100 \text{ km} = 225 \text{ km}
\]

\[
125 \text{ km left} \approx 93.75 \text{ km/hour}
\]
11. Find the standard form of the equation of the specified circle: center (+3,-2); solution point (-1,1).

\[ (x - 3)^2 + (y + 2)^2 = r^2 \]

\[ r = \text{distance from center to circle} \]

\[ = \sqrt{(3+1)^2 + (-2+1)^2} \]

\[ = \sqrt{16 + 1} \]

\[ = 5 \]

\[ (x - 3)^2 + (y + 2)^2 = 25 \]

In exercises 12-17 match the equation with its graph. Place the correct letter in the blank. [The graphs are labeled (a), (b), (c), (d), (e), and (f).] (2 pts ea)

12. \[ y = 1 - x \] \hspace{1cm} C
13. \[ y = x^2 - 2x \] \hspace{1cm} \alpha
14. \[ y = \sqrt{9 - x^2} \] \hspace{1cm} f
15. \[ y = 2\sqrt{x} \] \hspace{1cm} e
16. \[ y = x^3 - x + 1 \] \hspace{1cm} b
17. \[ y = |x| - 3 \] \hspace{1cm} d