NAME KEY
Instructor
Class Time

Math 4
Test 3
Fall 1998

You must *show all work* to receive full credit. Each question is worth the indicated value, for a total of 100 points possible. If you have any questions, please come to the front and ask or raise your hand.

1. (12 points) Describe the graph of each of the following equations as a circle, ellipse, parabola, hyperbola, or none of these:

a) $3x^2+6x-4y+12=0$ parabola

b) $3x^2+3y^2-6x+18y+10=0$ circle

c) $3x^2-2y^2+6x-8y+1=0$ hyperbola

d) $x^2+2x+4y^2+1=0$ ellipse

e) $2x^2-5y^2+4x-6=0$ hyperbola

f) $2x^2-3xy+4y^2-x+y-3=0$ neither

2. (6 points) Find a 6th degree polynomial whose *only* zeros occur at 1, 0, and 3.

$x^4(x-1)(x-3)$

$x^2(x-1)^2(x-3)^2$, etc.

3. (8 points) Rewrite after performing the division:

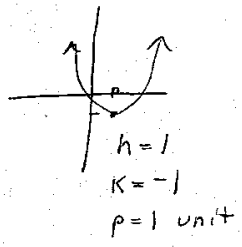
$$f(x)=\frac{6x^3-x^2-10x+1}{3x+1}= \quad 2x^2-x-3+\frac{4}{3x+1}$$

$$\begin{array}{r} 2x^2-x-3 \\ 3x+1\,\overline{)\,6x^3-x^2-10x+1} \\ \underline{6x^3+2x^2} \\ -3x^2-10x \\ \underline{-3x^2-x} \\ -9x+1 \\ \underline{-9x-3} \\ 4 \end{array}$$

4. (10 points) Find the equation of the parabola with vertex $(1,-1)$ and focus $(1,0)$.

$(x-h)^2 = 4p(y-k)$

$(x-1)^2 = 4(y+1)$



5. (10 points) Find all intercepts and asymptotes of $f(x) = \dfrac{x^2+x-2}{x-3}$, and sketch the graph of f. *Be sure to label your graph.*

$f(x) = \dfrac{(x+2)(x-1)}{x-3}$

y-intercept $(0, 2/3)$

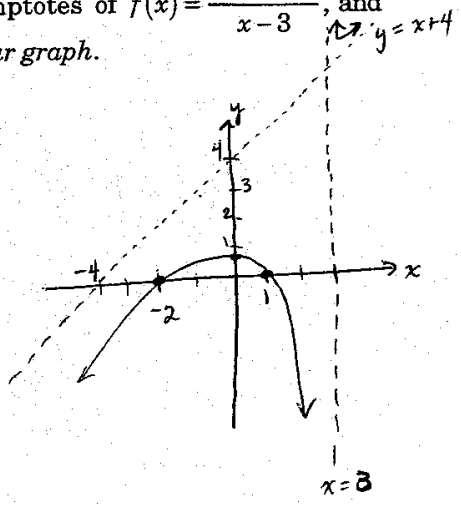
x-intercept $(-2,0), (1,0)$

vertical asymptote $x=3$

horizontal asymptote none

slant asymptote $y = x+4$

$$\begin{array}{r|rrr} 3 & 1 & 1 & -2 \\ & & 3 & 12 \\ \hline & 1 & 4 & 10 \end{array}$$



6. (6 points) List all the possible rational roots of $f(x) = 3x^5 + 2x^2 - 3x + 2$.

possible roots $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 3} \rightarrow \pm 1, \pm 2, \pm 1/3, \pm 2/3$

7. (8 points) Circle True or False:

T (F) $f(x) = x^3 + 2x^2 + x + 2$ has 3 negative real roots and no positive real roots.

(T) F On a graph of a parabola, the focus is located inside the parabolic curve.

(T) F An ellipse is a set of points, the sum of whose distances from two fixed points is constant.

T (F) $f(x) = \dfrac{x^2 + 2x - 3}{x + 3}$ has a vertical asymptote at $x = -3$.

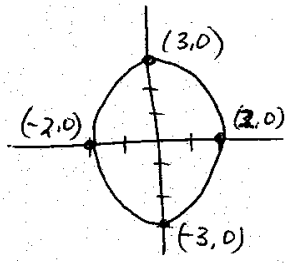
8. (8 points) Sketch the graph of $9x^2 + 4y^2 = 36$, labeling all vertices.

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$(0, \pm 3)$

$(\pm 2, 0)$

ellipse, center $(0,0)$



9. (8 points) Suppose $3i$ is a root of $f(x) = x^4 - 6x^3 + 14x^2 - 54x + 45$. Find *all* the remaining roots.

$-3i$ is also a root.

factors $(x - 3i)(x + 3i) = x^2 + 9$

$$x^2 + 9 \overline{)\, x^4 - 6x^3 + 14x^2 - 54x + 45}$$

$$-(x^4 \quad + 9x^2)$$

$$-6x^3 + 5x^2 - 54x$$

$$-(-6x^3 \quad - 54x)$$

$$5x^2 \quad + 45$$

$(x - 3i)(x + 3i)(x^2 - 6x + 5)$

$(x - 3i)(x + 3i)(x - 5)(x - 1)$

roots: $x = \pm 3i, 5, 1$

10. (8 points) Find the partial fraction decomposition of $\frac{2x^2+6x-11}{(x-3)(x+2)^2}$.

$$\frac{2x^2+6x-11}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$2x^2+6x-11 = Ax^2+4Ax+4A + B(x^2-x-6) + Cx-3C$$

$2 = A+B \longrightarrow B = 2-A$

$6 = 4A-B+C \longrightarrow 6 = 4A-2+A+C$

$-11 = 4A-6B-3C \qquad 8 = 5A+C$

$-11 = 4A-12+6A-3C \longrightarrow 1 = 10A-3C$

$16 = 10A+2C$

$15 = 5C$

$C = 3,\ A = 1,\ B = 1$

$$\frac{1}{x-3} + \frac{1}{x+2} + \frac{3}{(x+2)^2}$$

11. (6 points) Write $4x^2-5y^2-16x+30y-9=0$ in standard form.

$$4x^2-16x-5y^2+30y = 9$$

$$4(x^2-4x+4-4) - 5(y^2-6y+9-9) = 9$$

$$4(x-2)^2-16-5(y-3)^2-45 = 9$$

$$4(x-2)^2-5(y-3)^2 = 70$$

$$\frac{(x-2)^2}{5} - \frac{(y-3)^2}{4} = \frac{7}{2}$$

12. (10 points) Write $f(x)=x^4-3x^2-28$ as a product of factors irreducible over the rationals. As a product of factors irreducible over the real numbers. As a product of linear factors.

$$f(x) = (x^2-7)(x^2+4)$$

$$= (x+\sqrt{7})(x-\sqrt{7})(x+2i)(x-2i)$$

Irred. over rationals, $f(x) =$ $(x^2-7)(x^2+4)$

Irred. over reals, $f(x) =$ $(x+\sqrt{7})(x-\sqrt{7})(x^2+4)$

Linear factors, $f(x) =$ $(x+\sqrt{7})(x-\sqrt{7})(x+2i)(x-2i)$