November 11, 1997

Math 6
Exam 1

Name KEY
Section

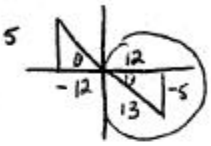
Show all work. Be neat.

Avg 78.7
High 93
Low 59

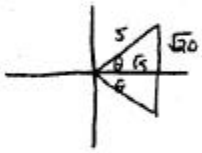
90-100	1
80-89	10
70-79	10
60-69	2
0-59	1

1. Determine the **EXACT** value of the following:

(4) (a) $\csc\left[\arctan\left(-\frac{5}{12}\right)\right] = \csc\theta = \frac{-13}{5}$



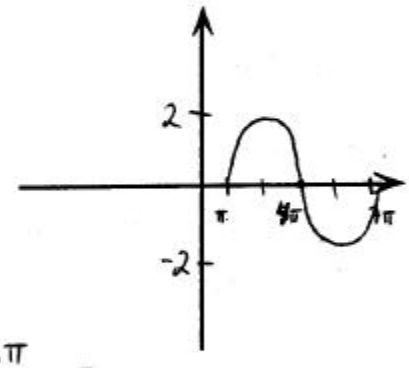
(4) (b) $\sin\left[\cos^{-1}\left(\frac{\sqrt{5}}{5}\right)\right] = \sin\theta = \frac{\sqrt{20}}{5} = \frac{2\sqrt{5}}{5}$



2. Sketch one period of $y = 2\sin\left(\frac{1}{3}x - \frac{\pi}{3}\right)$.

(8)

Start $\frac{1}{3}x - \frac{\pi}{3} = 0$
$x = \pi$
end $\frac{1}{3}x - \frac{\pi}{3} = 2\pi$
$\frac{1}{3}x = \frac{7\pi}{3}$
$x = 7\pi$
midpt $\frac{7\pi + \pi}{2} =$
period $= 2\pi/b = 6\pi$

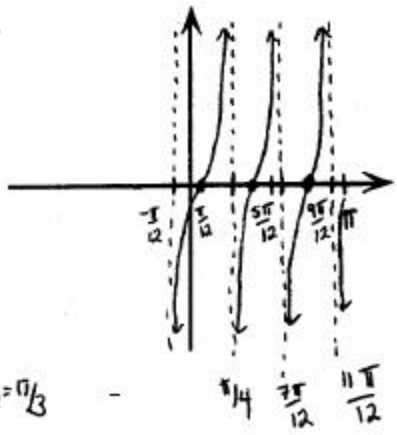


period: 6π

3. Sketch $y = \tan\left(3x - \frac{\pi}{4}\right)$ on $[0,\pi]$. Use dashed lines to indicate any asymptotes. Label asymptotes and x-intercepts clearly.

(8)

Start $3x - \pi/4 = -\pi/2$
$3x = -\pi/4$
$x = -\pi/12$
end $3x - \pi/4 = \pi/2$
$3x = 3\pi/4$
$x = \pi/4$
period $= \pi/b = \pi/3$
or $\pi/4 - (-\pi/12) = 4\pi/12 = \pi/3$



$-\pi/12$, 0, $\pi/6$, $\pi/4$
$\pi/3$

period: $\pi/3$

asymptotes $x = -\pi/12$
$x = \pi/4 = 3\pi/12$
$x = 7\pi/12$
$x = 11\pi/12$
$x = 15\pi/12 = 5\pi/4$

intercepts $x = \pi/12$
$x = 5\pi/12$
$x = 9\pi/12$
$x = 13\pi/12$

4. Mark answers clearly **TRUE** or **FALSE**.

(20)

? (a) $\sin(\sin^{-1} x) = x$ for all x true on domain... true where $\sin^{-1}x$ is defined, ie, if $-1 \le x \le 1$

F (b) $\sin^{-1}(\sin x) = x$ for all x only true if $-\pi/2 \le x \le \pi/2$

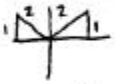
F (c) $\tan(-x) = \tan(x)$ $\tan(-x) = -\tan x$

F (d) $\csc^{-1} x = \sin x$

T? (e) $15°26'10'' = 15.436°$ $15° + \left(26' \cdot \frac{1°}{60'}\right) + \left(10'' \cdot \frac{1'}{60''} \cdot \frac{1°}{60'}\right) = 15° + .4\overline{3}° + .002\overline{7}°$ $= 15.436\overline{1}°$ close enough?

F (f) $100°$ is the complement of $-10°$ only positives have complements

F (g) 1 radian = 2π degrees $1\text{ rad} \cdot \frac{180°}{\pi\text{ rad}} = \frac{180°}{\pi}$

F (h) $\sin\theta = \frac{1}{2}$ implies $\cos\theta = \frac{\sqrt{3}}{2}$ 

F (i) for $y = \cos\left(2x - \frac{\pi}{4}\right)$, the phase shift is $\frac{\pi}{4}$ start $2x - \pi/4 = 0$ $2x = \pi/4$ $x = \pi/8$ = phase shift or p.s. = c/b =

F (j) $\sec(x)\csc(x) = 1$ $\frac{1}{\sin x \cos x}$

5. Convert 32.411° to degree-minute-second values.

(4)

$.411° \times \frac{60'}{1°} = 24.66'$

$.66' \times \frac{60''}{1'} = 39.6''$

$32°24'39.6''$

6. Let $\theta = 15°$.

(9) (a) Find the complement of θ.

$75°$

(b) Find the supplement of θ.

$165°$

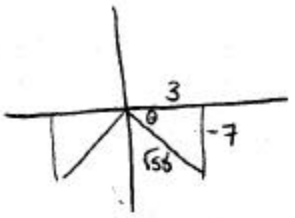
(c) Express θ in terms of π radians.

$\theta = 15° \times \frac{\pi}{180°} = \frac{\pi}{12}$ radians

7. Given that $\csc(\theta) = -\frac{\sqrt{58}}{7}$ and $\cos(\theta) > 0$, find $\sin\theta = \frac{-7}{\sqrt{58}}$

(6)

(a) $\tan(\theta) = -7/3$



$49 + x^2 = 58$

$x^2 = 9$

$x = 3$

(b) $\cos(\theta) = \frac{3}{\sqrt{58}}$

(c) $\sin(\theta) = -7/\sqrt{58}$

8. Calculate the linear speed in feet per minute of the tip of a 11-inch lawnmower blade when the engine is turning 1500 rpm (revolutions per minute).

(8)

$S = r\theta$

distance = radius · angle

can use 11 or $\frac{11}{2}$ for radius

1 revolution:

$S = 2\pi r = 22\pi$ inches

$\frac{1500 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ radians}}{1 \text{ rev}} = \frac{3000\pi \text{ radians}}{\text{min}} \times \frac{11 \text{ inches}}{\pi \text{ rad}} = 33000$

$\frac{1500 \text{ rev}}{\text{min}} \times \frac{22\pi \text{ in}}{1 \text{ rev}} = \frac{33000\pi \text{ inches}}{\text{min}} \cdot \frac{1 \text{ ft}}{12} = 2750\pi \text{ ft/min}$

want $\frac{S}{\text{time}} = \frac{\text{distance}}{\text{time}}$

≈ 8639.3798

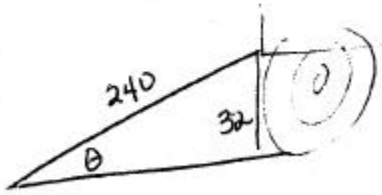
4319.69 if $\frac{11}{2}$

9. You are to use a 20 foot plank to create a ramp to a truck bed 2'8" high. What angle will the plank make with the (level) ground? Draw a picture of the situation and label known quantities.

(5)

$20\text{ft} \times \frac{12"}{1\text{ft}} = 240"$

$2'8" = 24" + 8" = 32"$



$\sin\theta = \frac{32}{240} = \frac{2}{15}$

$\theta = \sin^{-1}\frac{2}{15} = 7.66°$ or $.134$ radians

10. Suppose A and B are complementary angles.

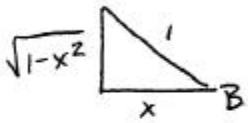
(9)

(a) Draw a right triangle and label angles A and B.

(b) If $\sin(A) = x$, what is $\sin(A)\cos(B)$ in terms of x?

$\sin A = x$
$\cos B = x$ $\quad \sin A \cos B = x^2$

(c) What is $\tan(B)$?



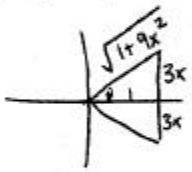
$\tan B = \frac{\sqrt{1-x^2}}{x}$

11. Write an algebraic expression equivalent to $\sec\left(\tan^{-1}(3x)\right)$. $= \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1+9x^2}} = \sqrt{1+$

(5)

$\theta = \tan^{-1} 3x$

$\tan\theta = 3x$

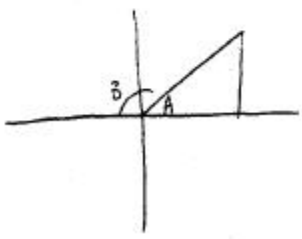


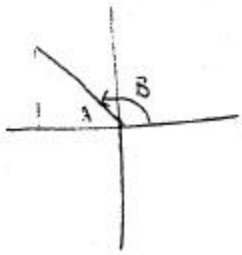
12. Calculate $\cos(81°)\cos(82°)\cos(83°)\cdot\ldots\cdot\cos(100°)$. $= 0$ since $\cos 90° = 0$

(5)

13. Suppose A and B are supplements. What can be said about the relationship between their respective cosines?

(5)





$\cos A = -\cos B$, x-values are opposites