Show all work. Be neat.

1. Determine the EXACT value of the following:

   (a) \( \csc[\arctan(-\frac{3}{12})] = \csc \theta = -\frac{13}{5} \)

   \( \theta \) is between \( -\pi/2 \) and \( \pi/2 \)

   (b) \( \sin[\cos^{-1}(\frac{3}{5})] = \sin \theta = \frac{\sqrt{20}}{5} = \frac{2\sqrt{5}}{5} \)

2. Sketch one period of \( y = 2\sin\left(\frac{1}{3} x - \frac{\pi}{3}\right) \).

   \( \text{Start } x = \frac{\pi}{3}, x = 0 \)
   \( \text{End } x = \frac{7\pi}{3}, x = 2\pi \)
   \( \text{Midpoint } \frac{7\pi + 2\pi}{2} \)
   \( \text{Period } \frac{2\pi}{\frac{1}{3}} = 6\pi \)

3. Sketch \( y = \tan(3x - \frac{\pi}{4}) \) on \([0, \pi]\). Use dashed lines to indicate any asymptotes. Label asymptotes and \( x \)-intercepts clearly.

   \( \text{Start } x = \frac{\pi}{4} = \frac{-\pi}{2} \)
   \( 3x = \frac{-\pi}{2} \)
   \( x = \frac{-\pi}{6} \)
   \( \text{End } x = \frac{\pi}{4} = \frac{\pi}{2} \)
   \( 3x = \frac{3\pi}{4} \)
   \( x = \frac{\pi}{4} \)
   \( \text{Period } \frac{\pi}{3} = \frac{\pi}{3} \)
   \( \text{or } \frac{\pi}{4} - (-\frac{\pi}{2}) = \frac{\pi}{12} = \frac{\pi}{12} \)

   Asymptotes: \( x = -\frac{\pi}{12}, \frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12} \)

   Intercepts: \( x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12} \)

   period: \( \frac{\pi}{3} \)
4. Mark answers clearly TRUE or FALSE.

(a) \(\sin(\sin^{-1} x) = x\) for all \(x\) with domain... true on domain...
(b) \(\sin^{-1}(\sin x) = x\) for all \(x\) only true if \(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\)
(c) \(\tan(-x) = \tan(x)\) \(\tan(-x) = -\tan x\)
(d) \(\csc^{-1} x = \sin x\)
(e) \(15^\circ26'10'' = 15.4365^\circ = (15^\circ + 0.4365^\circ) = 15^\circ + 0.4365^\circ \approx 15^\circ + 0.4365^\circ = 15.437^\circ\) close enough?
(f) 100° is the complement of -10° only positives have complements
(g) 1 radian = 2π degrees \(1\text{ rad} \cdot \frac{180^\circ}{\pi\text{ rad}} = \frac{180^\circ}{\pi}\)
(h) \(\sin \theta = \frac{1}{2}\) implies \(\cos \theta = \frac{\sqrt{3}}{2}\)
(i) for \(y = \cos(2x - \frac{\pi}{4})\), the phase shift is \(\frac{\pi}{4}\) start 2x - \(\frac{\pi}{4}\) = 0 or \(\frac{\pi}{4}\) = phase shift
(j) \(\sec(x)\csc(x) = 1\)

5. Convert 32.411° to degree-minute-second values.

\[
\begin{align*}
411^\circ \times \frac{60'}{1'} &= 24.66' \\
66' \times \frac{60'}{1'} &= 39.6''
\end{align*}
\]

\(32^\circ 24' 39.6''\)

6. Let \(\theta = 15^\circ\).

(a) Find the complement of \(\theta\).
\(75^\circ\)

(b) Find the supplement of \(\theta\).
\(165^\circ\)

(c) Express \(\theta\) in terms of \(\pi\) radians.
\(\theta = 15^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{12}\) radians
7. Given that \( \csc(\theta) = -\frac{\sqrt{58}}{7} \) and \( \cos(\theta) > 0 \), find \( \sin(\theta) = -\frac{7}{\sqrt{58}} \)

(a) \( \tan(\theta) = -\frac{7}{3} \)

(b) \( \cos(\theta) = \frac{3}{\sqrt{58}} \)

(c) \( \sin(\theta) = -\frac{7}{\sqrt{58}} \)

8. Calculate the linear speed in feet per minute of the tip of a 11-inch lawnmower blade when the engine is turning 1500 rpm (revolutions per minute).

\[
\text{distance} = \text{radius} \times \text{angle} \quad \text{rad/min}
\]

\[
1500 \text{ rev/min} \times \frac{2\pi \text{ radians}}{1 \text{ rev}} = 3000 \frac{2\pi \text{ radians}}{\text{min}}
\]

want \( s = \frac{\text{distance}}{\text{time}} \)

\[
= 3000 \frac{\text{rad}}{\text{min}} \times \frac{12 \text{ in}}{\pi \text{ rad}} = 33000 \text{ in/min}
\]

\[
\approx 8639.3798 \text{ ft/min}
\]

9. You are to use a 20 foot plank to create a ramp to a truck bed 2’8” high. What angle will the plank make with the (level) ground? Draw a picture of the situation and label known quantities.

\[
20 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} = 240 \text{ in}
\]

\[
2’8” = 24+8” = 32”
\]

\[
\sin \theta = \frac{32}{240} = \frac{2}{15}
\]

\[
\theta = \sin^{-1} \frac{2}{15} = 7.66^\circ \text{ or } 0.134 \text{ radians}
\]
10. Suppose $A$ and $B$ are complementary angles.
   
   (a) Draw a right triangle and label angles $A$ and $B$.
   
   
   (b) If $\sin(A) = x$, what is $\sin(A)\cos(B)$ in terms of $x$?
   
   
   \[
   \sin A = x \\
   \cos B = x \\
   \sin A \cos B = x^2
   \]
   
   (c) What is $\tan(B)$?
   
   
   \[
   \tan B = \frac{\sqrt{1-x^2}}{x}
   \]

11. Write an algebraic expression equivalent to $\sec(\tan^{-1}(3x))$.
   
   
   \[
   \sec \theta = \frac{1}{\cos \theta} = \frac{1}{\sqrt{1+9x^2}} = \sqrt{1+9x^2}
   \]

12. Calculate $\cos(81^\circ)\cos(82^\circ)\cos(83^\circ)\ldots\cos(100^\circ)$. = 0 since $\cos 90^\circ = 0$

13. Suppose $A$ and $B$ are supplements. What can be said about the relationship between their respective cosines?
   
   
   \[
   \cos A = -\cos B, \quad x-values \text{ are opposites}
   \]