Name KEY Exam 1 Instructor _____

Math 6 Fall 1998

You have 50 minutes to complete this test. You must show all work to receive full credit. Each question is worth the indicated value, for a total of 100 points possible. If you have any questions, please come to the front and ask.

1. Complete this chart, using exact values:

(20)

θ in radians	θ in degrees	sinθ	cosθ	tanθ	cscθ	secθ	cotθ
$-\frac{2\pi}{3}$	$-120°$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$	$-2/\sqrt{3}$	-2	$1/\sqrt{3}$
$5\pi/4$	$225°$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
π	$180°$	0	-1	0	undef	-1	undef
$\frac{\pi}{3}$	$60°$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$2/\sqrt{3}$	2	$1/\sqrt{3}$

2. Determine the **EXACT** value of the following:

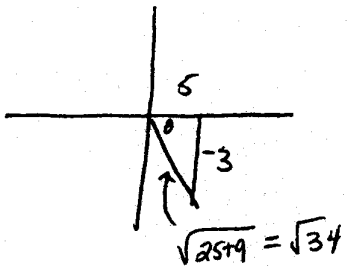
(6)

(a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$

Let $\theta = \arctan -3/5$

$\tan\theta = -3/5$ and $-\pi/2 < \theta < \pi/2$

$\sec\theta = \sqrt{34}/5$

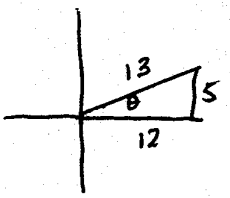


(b) $\cos\left[\sin^{-1}\frac{5}{13}\right]$

Let $\theta = \sin^{-1}\frac{5}{13}$

$\sin\theta = 5/13$ and $-\pi/2 < \theta < \pi/2$

$\cos\theta = 12/13$



3. Sketch one period of $y = \frac{1}{2}\sin\left(\frac{x}{2} - \frac{\pi}{3}\right)$.

(8)

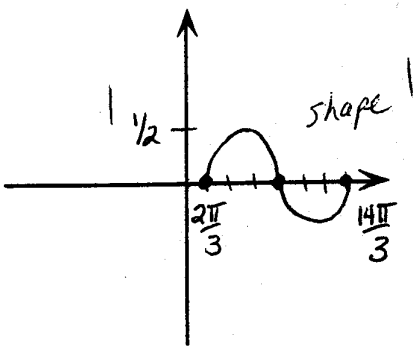
Start $\frac{x}{2} - \frac{\pi}{3} = 0$

$x = \frac{2\pi}{3}$

end $\frac{x}{2} - \frac{\pi}{3} = 2\pi$

$x = \frac{14\pi}{3}$

period $= \frac{12\pi}{3} = 4\pi$



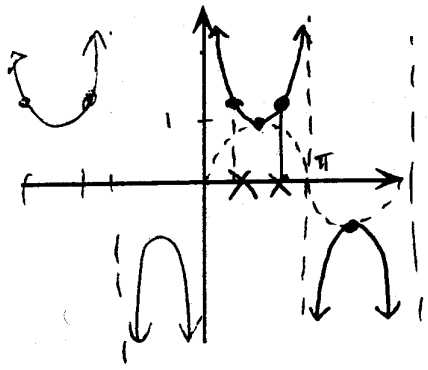
amplitude: 1/2 2

period: 4π 2

phase shift: 2π/3 2

4. (6) Graph the function and use it to solve the equation on the interval $[-2\pi, 2\pi]$.

$$\csc x = \sqrt{2}$$



$\sin x = \frac{1}{\sqrt{2}}$

$x = \pi/4, 3\pi/4$

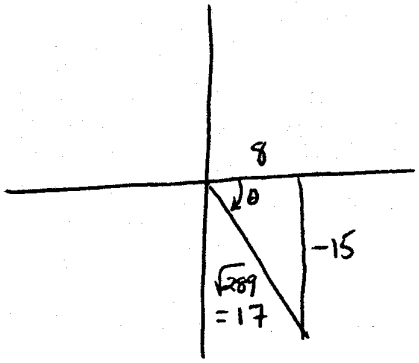
$-5\pi/4, -7\pi/4$

5. (6) Given that $\tan\theta = -\frac{15}{8}$ and $\sin\theta < 0$, find

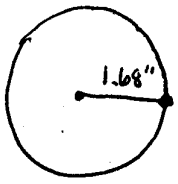
(a) $\tan(\theta) = -\frac{15}{8}$

(b) $\cos(\theta) = \frac{8}{17}$

(c) $\sin(\theta) = -\frac{15}{17}$



6. (8) The radius of the magnetic disk in a 3.5-inch diskette is 1.68 inches. Find the linear speed of a point on the circumference of the disk if it is rotating at a speed of 360 revolutions per minute.

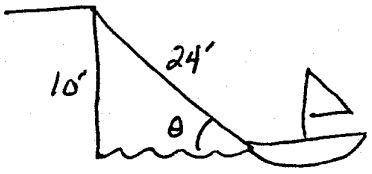


$$\frac{360 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ circumference}}{1 \text{ rev}} \times \frac{2\pi(1.68)''}{1 \text{ circ}}$$

$$= 1209.6\pi \text{ inches/min}$$

$$\approx 3800''/\text{min}$$

7. ( ) A boat is pulled in by means of a winch located on a dock 10 feet above the deck of the boat. Find the angle of elevation from the boat to the winch if the length of the rope from the winch to the boat is 24 feet. (Round your answer off to 2 decimal places.) Draw a picture of the situation and label known quantities.

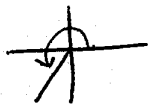


$\sin\theta = \frac{10}{24} = \frac{5}{12}$

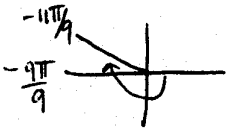
$\theta \approx .4298$ radians or 24.62°

8. (8) Determine the quadrant in which the angle (whose measure given in radians) terminates.

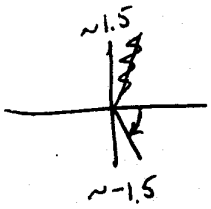
(a) $\frac{5\pi}{4}$ III



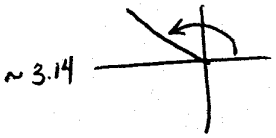
(b) $-\frac{11\pi}{9}$ II



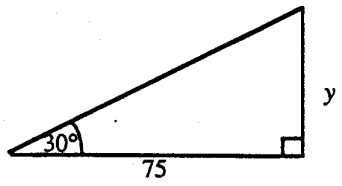
(c) -1 IV

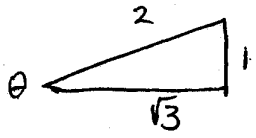


(d) 2.25 II



9. (6) Solve for y:





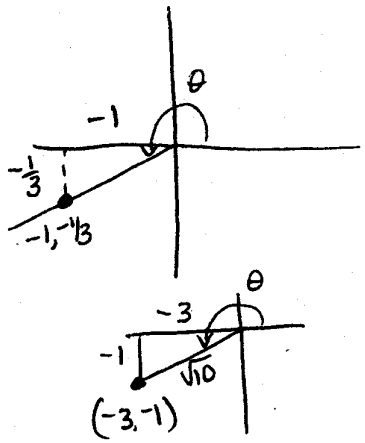
$\tan 30^\circ = y/75$

$y = 75 \tan 30^\circ$

$= 75\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{75}{\sqrt{3}} = \frac{75\sqrt{3}}{3} = 25\sqrt{3}$

10. The terminal side of θ lies on the line $y = \frac{1}{3}x$ in quadrant III. Find the values of the six trigonometric
(6) functions of θ.

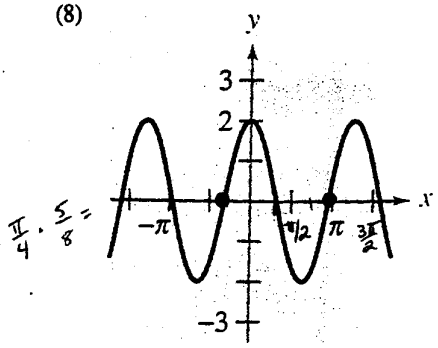


$\sin\theta = \frac{-1}{\sqrt{10}}$ $\quad \csc\theta = -\sqrt{10}$

$\cos\theta = \frac{-3}{\sqrt{10}}$ $\quad \sec\theta = -\sqrt{10}/3$

$\tan\theta = 1/3$ $\quad \cot\theta = 3$

11. Find a, b, and c for the function $y = a\sin(bx - c)$ so that the graph of f matches the figure.
(8)



$\frac{\pi}{4} \cdot \frac{5}{8} =$

Starts at $-\pi/4$ = phase shift $\rightarrow b(-\pi/4) - c = 0$

ends at π $\qquad c = \frac{-\pi}{4}b$

amplitude = $a = 2$ 2pts

$b\pi - c = 2\pi$

$b\pi - 2\pi = c = \frac{-\pi}{4}b$

$b\pi + \frac{\pi}{4}b = 2\pi$

$\frac{5}{4}b = 2$

$b = 8/5, \; c = \frac{-2}{5}\pi$

2pts 3pts 3pts

$y = 2\sin\left(\frac{8}{5}x + \frac{2}{5}\pi\right)$

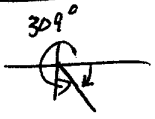
(OR) period $= \frac{5\pi}{4} = \frac{2\pi}{b}$ $\quad 5b = 8$ $\quad b = \frac{8}{5}$

phase shift $= -\pi/4 = c/b$

$c = \frac{-\pi}{4}b = -\pi/4\,(8/5) = \frac{-2}{5}\pi$

12. Find the following:

(2) (a) The reference angle of 309°: 51°

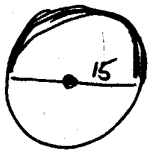


(2) (b) Both values of θ if $\cos\theta = -\frac{\sqrt{2}}{2}$: $\theta = 135°, 225°$ or $\frac{3\pi}{4}, \frac{5\pi}{4}$

$-\frac{\sqrt{2}}{2} = \frac{-1}{\sqrt{2}}$

(2) (c) The radian measure (as a multiple of π) of 315°: $315° \times \frac{\pi}{180°} = 7\pi/4$ radians

(4) (d) The length of the arc on a circle of radius 15 inches intercepted by a central angle of
180°: 15π"



$\theta = \frac{s}{r}$

$r\theta = s$

$15\pi = s$