November 19, 1998

Math 6
Exam 2

Name KEY
Section _____

1. (6 pts) Use the fundamental identities and factorization where necessary to simplify.

(a) $\dfrac{\cos^2 y}{1-\sin y} \cdot \dfrac{1+\sin y}{1+\sin y} = \dfrac{\cos^2 y(1+\sin y)}{(1-\sin^2 y)} = 1+\sin y$

$1+\sin y$

(b) $\sin^2 x \sec^2 x - \sin^2 x = \sin^2 x(\sec^2 x - 1)$

$= \sin^2 x \tan^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\sin^2 x \tan^2 x$

2. (6 pts) Use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \frac{\pi}{2}$.

$\sqrt{x^2-4}, \; x = 2\sec\theta$

$\sqrt{4\sec^2\theta - 4} = 2\sqrt{\sec^2\theta - 1}$

2 pts

$= 2\sqrt{\tan^2\theta}$

$= 2|\tan\theta|$

$= 2\tan\theta$ betw 0 & $\pi/2$

3. (10 pts) Match the trigonometric expression with one of the following simplified expressions (A-J). Letters can be used more than once.

(a) $\dfrac{\cot x}{\csc x} =$ D $\cos x$ $\quad \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$

(b) $(\sec x + 1)(\sec x - 1) =$ H $\quad \sec^2 x - 1 = \tan^2 x$

(c) $\cos\left(\frac{\pi}{2} - x\right)\sec x =$ B $\quad \dfrac{\sin x}{\cos x} = \tan x$

(d) $\tan^4 x + 2\tan^2 x + 1 =$ C $\quad (\tan^2 x + 1)^2 = \sec^4 x$

(e) $\dfrac{\sin(-x)}{\cos(-x)} =$ F $\quad -\dfrac{\sin x}{\cos x}$

(f) $\cos x \tan x =$ A $\quad \sin x$

A. $\sin x$
B. $\tan x$
C. $\sec^4 x$
D. $\cos x$
E. $\sec^2 x$
F. $-\tan x$
G. $\cot x$
H. $\tan^2 x$
I. $-\tan^2 x$
J. 1

4. (30 pts) Verify the following identities. Only work from 1 side. Work must be neat!

(a) $\dfrac{\cos\theta\cot\theta}{1-\sin\theta} - 1 = \csc\theta$

$$\frac{\cos\theta\cot\theta}{1-\sin\theta} = \frac{\frac{\cos^2\theta}{\sin\theta}}{1-\sin\theta}\,\frac{(1+\sin\theta)}{(1+\sin\theta)} - 1$$

$$= \frac{\frac{\cos^2\theta(1+\sin\theta)}{\sin\theta}}{1-\sin^2\theta} - 1$$

$$= \frac{1+\sin\theta}{\sin\theta} - 1$$

$$= \frac{1+\sin\theta-\sin\theta}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$$

-5 if got about halfway

(b) $\sin x\left(1-2\cos^2 x+\cos^4 x\right) = \sin^5 x$

$$\sin x(1-2\cos^2 x+\cos^4 x) = \sin x(1-\cos^2 x)(1-\cos^2 x)$$

if used $\cos^2 x - 1 \downarrow \sin^2 x$ then -2

$$= \sin x \sin^2 x \sin^2 x$$

$$= \sin^5 x$$

(c) $\csc x - \sin x = \cos x\cot x$

$$\csc x - \sin x = \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x}$$

$$= \cos x\,\frac{\cos x}{\sin x}$$

$$= \cos x\cot x$$

5. (28 pts) Find all solutions in the interval $[0,2\pi)$ of the following trigonometric equations. Show all work! Give exact solutions.

(a) $2\sin^2 x = 2+\cos x$

$2(1-\cos^2 x) = 2+\cos x$

$2-2\cos^2 x = 2+\cos x$

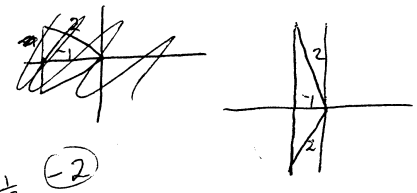
$-2\cos^2 x + \cos x = 0$ (lost the −2)

$2\cos x(2\cos x+1)=0$

$\cos x = 0$ or $\cos x = -\frac{1}{2}$ ($\cos x = \frac{1}{2}$ (−2))

$x = \begin{cases} \pi/2 + 2n\pi \\ 3\pi/2 + 2n\pi \end{cases}$ $x = \begin{cases} 2\pi/3 + 2n\pi \\ 4\pi/3 + 2n\pi \end{cases}$

in $[0,2\pi)$, $x = \pi/2,\ 3\pi/2,\ 2\pi/3,\ 4\pi/3$



(b) $2\sec^2 x + \tan^2 x - 3 = 0$

$2(1+\tan^2 x) + \tan^2 x - 3 = 0$

$3\tan^2 x - 1 = 0$

$\tan^2 x = 1/3$ (−2)

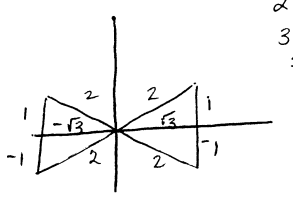
$\tan x = \pm 1/\sqrt{3}$ (wrong ±'s (did $\pi/3$) (−2))

$2\sec^2 x + \sec^2 x - 1 - 3 = 0$

$3\sec^2 x = 4$

$\sec^2 x = 4/3$

$\cos x = \pm\sqrt{3}/2$



$x = \begin{cases} \pi/6 + 2n\pi \\ 5\pi/6 + 2n\pi \\ 7\pi/6 + 2n\pi \\ 11\pi/6 + 2n\pi \end{cases}$ or $x = \begin{cases} \pi/6 + n\pi \\ 5\pi/6 + n\pi. \end{cases}$

in $[0,2\pi)$, $x = \pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6$

(c) $\cot x \cos^2 x = 2\cot x$

$\cot x \cos^2 x - 2\cot x = 0$ (3 pts if got this)

$\cot x(\cos^2 x - 2) = 0$

$\cot x = 0$ (−3 if left out) $\quad \cos x = \pm\sqrt{2}$ none (−3 if didn't realize no solns)

$\frac{\cos x}{\sin x} = 0$

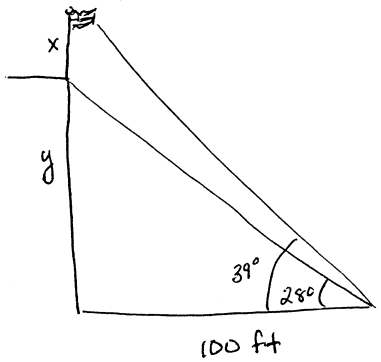
said $\cot x = 0$ but didn't get x

$\cos x = 0$

$x = \begin{cases} \pi/2 + 2n\pi \\ 3\pi/2 + 2n\pi \end{cases}$

in $[0,2\pi)$, $x = \pi/2,\ 3\pi/2$

6. (10 pts) A flagpole is mounted on the front of a library's roof. From a point 100 feet in front of the library, the angle of elevation to the base of the flagpole and the top of the flagpole are 28° and 39°. Find the height of the flagpole and the height of the library. (Round answers to the nearest whole number.)



$\tan 28° = y/100$

$y = 100 \tan 28° \approx 53.17$ ft

$\tan 39° = \frac{x+y}{100}$

$y + x = 100 \tan 39° \approx 80.98$ ft

$x \approx 80.98 - 53.17$

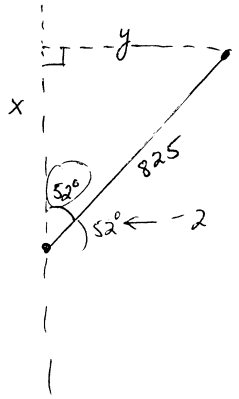
≈ 27.81 ft

used sin, cot instead of tan -3

flagpole ≈ 28 ft

building ≈ 53 ft

7. (10 pts) An airplane flying at 550 miles per hour has a bearing of N52°E. After flying 1.5 hours, how far north and how far east will the plane have traveled from its point of departure?



$\frac{550 \text{ miles}}{\text{hr}} \cdot 1.5 \text{ hrs} = 550 + 275 = 825$ miles

$\sin 52° = y/825$ $\quad y = 825 \sin 52°$

≈ 650.1 miles east

$\cos 52° = x/825$ $\quad x = 825 \cos 52°$

≈ 507.9 miles North

-1 put ft

-1 mixed up

650.1 miles East

507.9 miles North