December 8, 1998

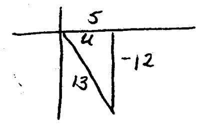
Math 6
Exam 3

Name KEY
Section

1. (10 pts) Find the exact value of the trigonometric function given that $\cos u = \frac{5}{13}$, $\sin v = -\frac{3}{5}$ and both u and v are in quadrant IV.

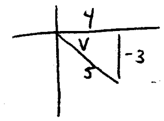
(a) $\csc(u-v) = \dfrac{1}{\sin(u-v)} = \dfrac{1}{\sin u \cos v - \cos u \sin v}$

$= \dfrac{1}{\left(\frac{-12}{13}\right)\left(\frac{4}{5}\right)-\left(\frac{5}{13}\right)\left(\frac{-3}{5}\right)} = \dfrac{65}{-48+15} = \dfrac{-65}{33}$



(b) $\cot(u+v) = \dfrac{1}{\tan(u+v)} = \dfrac{1-\tan u \tan v}{\tan u + \tan v} =$

$= \dfrac{1-\left(\frac{-12}{5}\right)\left(\frac{-3}{4}\right)}{\frac{-12}{5}-\frac{3}{4}} = \dfrac{1-\frac{36}{20}}{\frac{-48-15}{20}} = \dfrac{-16}{-63} = \dfrac{16}{63}$



2. (10 pts) Find all solutions in the interval $[0, 2\pi)$.

$\sin\left(x+\frac{\pi}{4}\right)-\sin\left(x-\frac{\pi}{4}\right)=1$

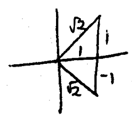
$\left[\sin x\left(\frac{1}{\sqrt{2}}\right)+\cos x\left(\frac{1}{\sqrt{2}}\right)\right]-\left[\sin x\left(\frac{1}{\sqrt{2}}\right)-\cos x\left(\frac{1}{\sqrt{2}}\right)\right]=1$

(-5) if got to $2\cos x \sin \pi/4 = 1$

$\frac{2}{\sqrt{2}}\cos x = 1$

$\cos x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$x = \pi/4,\ \cancel{-\pi/4}\ \ 7\pi/4$



3. (8 pts) Find the exact values of $\sin\left(\frac{u}{2}\right)$ and $\cos\left(\frac{u}{2}\right)$ given

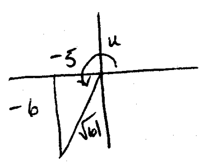
$\tan u = \frac{6}{5} \quad \pi < u < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4}$ 2nd quadrant

(-4) if got one sign wrong, didn't fill in u in formula

$\sin\left(\frac{u}{2}\right) = +\sqrt{\dfrac{1-\cos u}{2}}$

$= \sqrt{\dfrac{1+5/\sqrt{61}}{2}} = \sqrt{\dfrac{\sqrt{61}+5}{2\sqrt{61}}} = \sqrt{\dfrac{61+5\sqrt{61}}{122}}$



$\cos\left(\frac{u}{2}\right) = -\sqrt{\dfrac{1+\cos u}{2}}$

$= -\sqrt{\dfrac{1-5/\sqrt{61}}{2}} = -\sqrt{\dfrac{\sqrt{61}-5}{2\sqrt{61}}}$

-2 choose wrong sign
+2 picture

4. (12 pts) Rewrite the expression in terms of the first power of cosine.

$$\sin^2 x\cos^2 x = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) = \frac{1}{4}\left(1-\cos^2 2x\right)$$

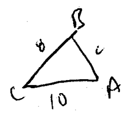
-8 if got to

$$= \underbrace{\frac{1}{4}\left(1-\frac{1+\cos 4x}{2}\right)}_{-2?} = \frac{1}{4}-\frac{1}{8}-\frac{1}{8}\cos 4x$$

$$= \frac{1}{8}-\frac{1}{8}\cos 4x$$

5. (12 pts) Solve a triangle with the given information. If 2 solutions exist find both. If no solution exists state why.

$A = 46°$ $a = 8$ $b = 10$



$$\frac{\sin B}{10} = \frac{\sin 46°}{8}$$

$$\sin B = \frac{5}{4}\sin 46° \approx 0.8992$$

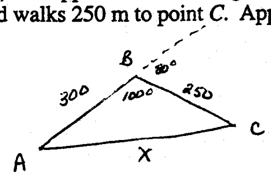
-4 didn't get 2nd value for B

$B \approx 64°$, $C \approx 70°$ OR $B \approx 116°$, $C \approx 18°$

$$\frac{c}{\sin 70°} = \frac{8}{\sin 46°} \qquad \frac{c}{\sin 18°} = \frac{8}{\sin 46°}$$

$c \approx 10.45$ $\qquad$ $c \approx 3.44$

6. (12 pts) To approximate the length of a marsh, a surveyor walks 300 m from point A to point B, then turns 80° and walks 250 m to point C. Approximate the length from point A to point C.



$$x^2 = 300^2 + 250^2 - 2(300)(250)\cos 100°$$
$$= 90000 + 62500 - 150000\,(-.1736)$$
$$\approx 178547.227$$
$$x \approx 422.5 \text{ m}$$

7. (12 pts) Convert to trigonometric form and then perform the indicated operation. Leave your answer in trigonometric form.

(a) $(\sqrt{3}-i)(2-2\sqrt{3}i) = 2(\cos(-\pi/6) + i\sin(-\pi/6)) \cdot 4(\cos(\frac{4\pi}{3}) + i\sin(-\pi/3))$

$= 8(\cos(-\pi/2) + i\sin(-\pi/2))$

OR $= 8(\cos 3\pi/2 + i\sin 3\pi/2)$

$r = \sqrt{3+1} = 2$, $\tan\theta = \frac{-1}{\sqrt{3}}$, $\theta = -\pi/6$

$r = \sqrt{4+12} = 4$, $\tan\theta = -\sqrt{3}$

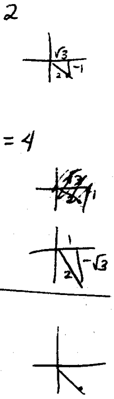
(b) $(1-i)^7 = \left[\sqrt{2}(\cos(-\pi/4) + i\sin(-\pi/4))\right]^7$

$= (\sqrt{2})^7(\cos(-7\pi/4) + i\sin(-7\pi/4))$

$= (\sqrt{2})^7(\cos \pi/4 + i\sin \pi/4)$

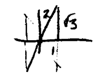
$= 8\sqrt{2}(\cos \pi/4 + i\sin \pi/4)$

$r = \sqrt{2}$, $\tan\theta = -1$, $\theta = -\pi/4$



8. (12 pts) Find all possible roots to $z^6 - 1 = 0$. 6th roots of $1 = \cos 0 + i \sin 0$

$$\text{root} = \cos \frac{0 + 2k\pi}{6} + i \sin \frac{0 + 2k\pi}{6}$$



If $k=0$, root $= \cos 0 + i \sin 0 = \boxed{1}$

If $k=1$, root $= \cos \pi/3 + i \sin \pi/3 = \boxed{\frac{1}{2} + i \frac{\sqrt{3}}{2}}$

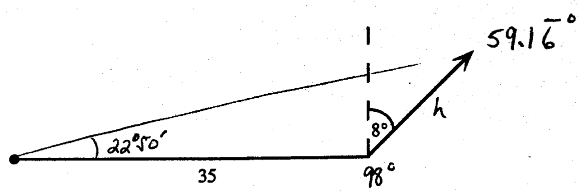
If $k=2$, root $= \cos 2\pi/3 + i \sin 2\pi/3 = \boxed{-\frac{1}{2} + i \sqrt{3}/2}$

If $k=3$, root $= \cos \pi + i \sin \pi = \boxed{-1}$

If $k=4$, root $= \cos 4\pi/3 + i \sin 4\pi/3 = \boxed{-\frac{1}{2} - i \frac{\sqrt{3}}{2}}$

If $k=5$, root $= \cos 5\pi/3 + i \sin 5\pi/3 = \boxed{\frac{1}{2} - i \frac{\sqrt{3}}{2}}$

9. (12 pts) Because of prevailing winds, a tree grew so that it was leaning 8° from the vertical. At a point 35 m from the base of the tree, the angle of elevation to the top of the tree is 22°50′. Find the height of the tree.



$$22°50' = 22° + \left(50' \times \frac{1°}{60'}\right) = 22.8\bar{3}°$$

$$\frac{h}{\sin 22.8\bar{3}°} = \frac{35}{\sin 59.1\bar{6}°}$$

$$h = 35 \frac{\sin 22.8\bar{3}°}{\sin 59.1\bar{6}°}$$

$h \approx 15.82\text{m}$