NAME KEY

Math 6
Test 3
Winter 1998

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Each question is worth the indicated ~~value~~, for a total of 100 points possible. You may also earn 5 bonus points from the bonus problem. If you have any questions, please come to the front and ask.

1. (12 points) Find the *areas* of the triangles with the following sides and angles (answers in decimal form are fine):

   a) a = 80, b = 51, c = 113.

   $s = \frac{a+b+c}{2} = 122$

   $\text{area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{122(42)(71)(9)} = \sqrt{3274236}$

   ≈ 1809.5

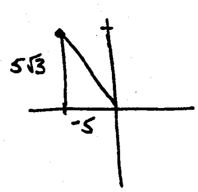
   b) A = 71°, b = 10, c = 19.

   $\text{area} = \frac{1}{2} bc \sin A$

   $= \frac{1}{2}(10)(19)(\sin 71°)$

   ≈ 89.8

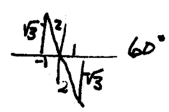
2. (5 points) Write $-5+5\sqrt{3}\,i$ in trigonometric form, and also plot this point on the complex plane. Use *exact* values, not decimal approximations.



$r = \sqrt{25+75} = 10$

$\tan\theta = \frac{5\sqrt{3}}{-5} = -\sqrt{3}$

$\theta = 120°$ or $2\pi/3$



$-5+5\sqrt{3}\,i = 10(\cos 2\pi/3 + i\sin 2\pi/3)$

$= 10(\cos 120° + i\sin 120°)$

3. (10 points) Find *all* solutions of $2\sin^2\left(x+\frac{\pi}{2}\right)=1$. Your answers should be *exact*.

$\sin^2(x+\pi/2) = 1/2 \longrightarrow$

$\sin(x+\pi/2) = \pm 1/\sqrt{2} \longrightarrow \cancel{\sin x \cos \pi/2} + \cos x \sin \pi/2 = \pm 1/\sqrt{2}$

$\cos x = \pm 1/\sqrt{2}$

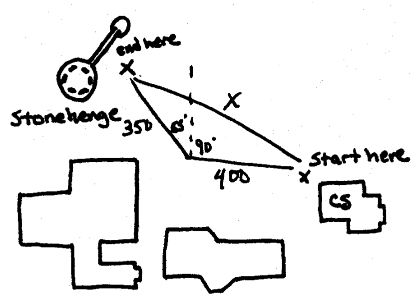
$x = \pi/4 + n\pi/2$

$$x + \pi/2 = \begin{cases} \pi/4 + 2n\pi \\ 3\pi/4 + 2n\pi \\ 5\pi/4 + 2n\pi \\ 7\pi/4 + 2n\pi \end{cases}$$

$x + \pi/2 = \pi/4 + n\pi/2$

$x = -\pi/4 + n\pi/2 = \pi/4 + n\pi/2$

4. (7 points) To get from the Computer Science Building to UMR's Stonehenge, you can walk due west 400 ft, then turn to a bearing of N55°W, and walk 350 ft to Stonehenge. Assume these are all straight line paths. How far is it in a straight line from the CS building to Stonehenge? (a decimal approximation is fine)



$x^2 = 400^2 + 350^2 - 2(400)(350)\cos 145°$

$= 282500 -$

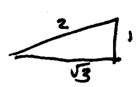
$= \cancel{511862.57}$

$x \approx 715.4 \text{ ft}$

5. (12 points) Find *exact* values for

a) $\csc\frac{\pi}{12} = \dfrac{1}{\sin(\pi/3 - \pi/4)} = \dfrac{1}{\sin\pi/3\cos\pi/4 - \sin\pi/4\cos\pi/3} = \dfrac{1}{\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}}$

$= \dfrac{2\sqrt{2}}{\sqrt{3}-1} \cdot \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{2\sqrt{6}+2\sqrt{2}}{3-1} = \sqrt{6}+\sqrt{2}$



$\dfrac{\sqrt{3}-3}{3+\sqrt{3}} \cdot \dfrac{\sqrt{3}-3}{\sqrt{3}-3} = \dfrac{12-6\sqrt{3}}{-6} = -2+\sqrt{3}$

$\dfrac{4}{\sqrt{6}-\sqrt{2}} = \dfrac{4(\sqrt{6}+\sqrt{2})}{6-2} = \sqrt{6}+\sqrt{2}$

b) $\tan(-15°) = \tan(30° - 45°)$

$= \dfrac{\sin 30°\cos 45° - \sin 45°\cos 30°}{\cos 30°\cos 45° + \sin 30°\sin 45°} = \dfrac{\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}}{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}}$

$= \dfrac{1-\sqrt{3}}{1+\sqrt{3}} \cdot \dfrac{1-\sqrt{3}}{1-\sqrt{3}} = \dfrac{1-2\sqrt{3}+3}{1-3} = \dfrac{4-2\sqrt{3}}{-2} = \sqrt{3}-2$

6. (10 points) Verify the following identity (used in calculus):

$$\frac{\sin(x+h)-\sin x}{h}=\frac{\sin x(\cos h-1)}{h}+\frac{\cos x \sin h}{h}$$

$$\frac{\sin(x+h)-\sin x}{h} = \frac{\sin x \cosh + \cos x \sin h - \sin x}{h}$$

$$= \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h}$$

7. (10 points) Calculate the following, and write your answers in standard form (*exact* answers, please):

a) $$\frac{12\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}{4\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)} = 3\left(\cos \pi/4 + i\sin \pi/4\right) = \frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}}i$$

b) $$\left[3\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)\right]^4 = 81\left(\cos \pi/3 + i\sin \pi/3\right) = 81\left(\tfrac{1}{2}+\tfrac{\sqrt{3}}{2}i\right)$$

$$= \frac{81}{2} + \frac{81\sqrt{3}}{2}i$$

8. (14 points) Solve the triangles with sides and angles as follows (decimal approximations are fine; be sure to explain/show work):

a) A = 95°, a = 6, b = 8.

not possible, A is biggest angle so a should be longest side!

b) A = 40°, a = 9, b = 12.

$$\frac{\sin B}{12} = \frac{\sin 40^\circ}{9}$$

$\sin B = .857$

$B \approx 59^\circ$

$C \approx 81^\circ$

$\frac{c}{\sin 81^\circ} = \frac{9}{\sin 40^\circ}$

$c \approx 13.8$

or

$B \approx 180^\circ - 59^\circ \approx 121^\circ$

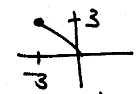
$C \approx 19^\circ$

$\frac{c}{\sin 19^\circ} = \frac{9}{\sin 40^\circ}$

$c \approx 4.56$

9. (10 points) Find all fourth roots of $-3+3i$. You may leave your answers in trigonometric form, but they should be *exact*.

$z=-3+3i$, $r=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$, $\theta=3\pi/4$



$z=3\sqrt{2}\,(\cos 3\pi/4+i\sin 3\pi/4)$

if $u^4=z$, then $u=\sqrt[4]{3\sqrt{2}}\left(\cos\frac{3\pi/4+2k\pi}{4}+i\sin\frac{3\pi/4+2k\pi}{4}\right)$

-2 not $\sqrt[4]{\sqrt{18}}$
-2 not filling in k
-2 got $\pi/4$, not $3\pi/4$
-3 ignored θ

$$u=\begin{cases}\sqrt[4]{3}\sqrt[8]{2}\,(\cos 3\pi/16+i\sin 3\pi/16)\\ \sqrt[4]{3}\sqrt[8]{2}\,(\cos 11\pi/16+i\sin 11\pi/16)\\ \sqrt[4]{3}\sqrt[8]{2}\,(\cos 19\pi/16+i\sin 19\pi/16)\\ \sqrt[4]{3}\sqrt[8]{2}\,(\cos 27\pi/16+i\sin 19\pi/16)\end{cases}$$

10. (10 points) Find all solutions of $\cos 4x+\cos 2x=0$ in $[0,2\pi)$. Give *exact* answers.

-2 used 1 formula, quit
-5 used several, got nowhere
-1 sign error
-2 messed up end, not all the solns

$\cos 4x+\cos 2x=0$

$\cos^2 2x-\sin^2 2x+\cos 2x=0$

$\cos^2 2x-(1-\cos^2 2x)+\cos 2x=0$

$2\cos^2 2x+\cos 2x-1=0$

$(\cos 2x+1)(2\cos 2x-1)=0$

$\cos 2x=-1$ $\qquad$ $\cos 2x=1/2$

$2x=\pi+2n\pi$ $\qquad$ $2x=\begin{cases}\pi/3+2n\pi\\ -\pi/3+2n\pi\end{cases}$

$x=\pi/2+n\pi$ $\qquad$ $x=\pm\pi/6+n\pi$

$x=\pi/2,\ 3\pi/2$ $\qquad$ $x=\pi/6,\ 7\pi/6,\ 5\pi/6,\ 11\pi/6$

$u=3x$
$v=x$

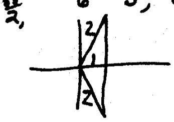
$\frac{1}{2}(\cos 4x+\cos 2x=0)$

$\frac{1}{2}(\cos(u+v)+\cos(u-v))=0$

$\cos u\cos v=0$

$\cos x\cos 3x=0$

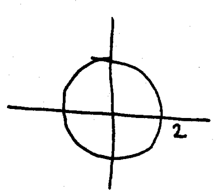
$\frac{\pi}{2},\frac{3\pi}{2}$, $\quad \frac{\pi}{6}+\frac{2n\pi}{3}$, $\quad \frac{3\pi}{8}+\frac{2n\pi}{3}$



BONUS Question (5 points)

In rectangular coordinates, we are used to using the variables x and y in equations. In polar (trigonometric) equations, however, we use the variables r and θ. Using a polar graph (the complex plane), graph the following functions:

a) $r=2$



b) $\theta = 45°$