You have 50 minutes to complete this test. You must show all work to receive full credit. Each question is worth the indicated value, for a total of 100 points possible. You may also earn 5 bonus points from the bonus problem. If you have any questions, please come to the front and ask.

1. (12 points) Find the areas of the triangles with the following sides and angles (answers in decimal form are fine):

   a) \( a = 80, b = 51, c = 113. \)
   \[
   s = \frac{a+b+c}{2} = 122
   \]
   \[
   \text{area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{122(42)(71)(9)} = \sqrt{3274284} \\
   \approx 1809.5
   \]

   b) \( A = 71^\circ, b = 10, c = 19. \)
   \[
   \text{area} = \frac{1}{2}bc \sin A \\
   = \frac{1}{2}(10)(19)(\sin 71^\circ) \\
   \approx 89.8
   \]

2. (5 points) Write \(-5+5\sqrt{3}i\) in trigonometric form, and also plot this point on the complex plane. Use exact values, not decimal approximations.

   \[
   r = \sqrt{25+75} = 10
   \]
   \[
   \tan \theta = \frac{5\sqrt{3}}{-5} = -\sqrt{3} \\
   \theta = 120^\circ \text{ or } \frac{5\pi}{3}
   \]
   \[
   -5+5\sqrt{3}i = 10 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) \\
   = 10 \left( \cos 120^\circ + i \sin 120^\circ \right)
   \]
3. (10 points) Find all solutions of \(2 \sin^2\left(\frac{x + \frac{\pi}{2}}{2}\right) = 1\). Your answers should be exact.

\[
\sin^2\left(\frac{x + \frac{\pi}{2}}{2}\right) = \frac{1}{2} \quad \rightarrow \quad \sin\left(\frac{x + \frac{\pi}{2}}{2}\right) = \pm \sqrt{\frac{1}{2}}
\]

\[
\sin\left(\frac{x + \frac{\pi}{2}}{2}\right) = \pm \sqrt{\frac{1}{2}} \quad \rightarrow \quad \sin\frac{x + \frac{\pi}{2}}{2} \pm \cos\frac{x + \frac{\pi}{2}}{2} = \pm \sqrt{\frac{1}{2}}
\]

\[
\cos x = \pm \sqrt{\frac{1}{2}}
\]

\[
x = \pm \frac{\pi}{4} + n\frac{\pi}{2}
\]

4. (7 points) To get from the Computer Science Building to UMR's Stonehenge, you can walk due west 400 ft, then turn to a bearing of N65°W, and walk 350 ft to Stonehenge. Assume these are all straight line paths. How far is it in a straight line from the CS building to Stonehenge? (a decimal approximation is fine)

\[
x^2 = 400^2 + 350^2 - 2(400)(350)\cos 65°
\]

\[
x = \sqrt{512,425.7} 
\]

\[
x \approx 715.4 \text{ ft}
\]

5. (12 points) Find exact values for

a) \(\csc \frac{\pi}{12} = \frac{1}{\sin\left(\frac{\pi}{12}\right)} = \frac{1}{\sqrt{\frac{1}{2}} - \frac{\sqrt{3}}{2}}\)

\[
= \frac{1}{\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{2} - \frac{\sqrt{3}}{2}}
\]

b) \(\tan(-15°) = \frac{\sin(-15°)}{\cos(-15°)} = -\frac{\sin 75°}{\cos 75°} = \frac{\sin 30° \cos 45° - \sin 45° \cos 30°}{\cos 30° \cos 45° + \sin 30° \sin 45°}
\]

\[
= \frac{\frac{1}{2} \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{1}{2} \frac{\sqrt{2}}{2}}
\]

\[
= \frac{\frac{1}{2} \frac{1}{2} - \frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2} + \frac{1}{4} \frac{\sqrt{2}}{2}} = \frac{\frac{1}{2} \frac{1}{2} - \frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2} + \frac{1}{4} \frac{\sqrt{2}}{2}}
\]

\[
= \frac{\frac{1}{4} - \frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2} + \frac{1}{4} \frac{\sqrt{2}}{2}} = \frac{1}{2} \frac{1}{2} - \frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2}
\]

\[
= \frac{1}{2} \frac{1}{2} - \frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2} = \frac{1}{2} \frac{1}{2} - \frac{\sqrt{3}}{4} \frac{\sqrt{2}}{2}
\]

\[
= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} = \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2}
\]
6. (10 points) Verify the following identity (used in calculus):

\[
\frac{\sin(x+h) - \sin x}{h} = \frac{\sin x (\cos h - 1)}{h} + \frac{\cos x \sin h}{h}
\]

\[
= \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}
\]

\[
= \frac{\sin x (\cosh - 1) + \cos x \sinh}{h}
\]

7. (10 points) Calculate the following, and write your answers in standard form (exact answers, please):

a) \[
\frac{12 \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)}{4 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)} = \frac{3}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i} = \frac{3(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i)}{2}
\]

b) \[
\left[ 3 \left( \cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) \right]^4 = \frac{3}{2} \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) = \frac{9}{2} \left( 1 + i \frac{\sqrt{3}}{2} \right)
\]

8. (14 points) Solve the triangles with sides and angles as follows (decimal approximations are fine; be sure to explain/show work):

a) \( A = 95^\circ, a = 6, b = 8. \)

Not possible, \( A \) is biggest angle so \( a \) should be longest side.

b) \( A = 40^\circ, a = 9, b = 12. \)

\[
\frac{\sin B}{12} = \frac{\sin 59^\circ}{9}
\]

\[
\sin B = 0.857
\]

\[
B \approx 59^\circ, C \approx 81^\circ
\]

\[
C \approx 13.8
\]
9. (10 points) Find all fourth roots of $-3 + 3i$. You may leave your answers in trigonometric form, but they should be exact.

\[
\begin{align*}
z &= -3 + 3i, \\
r &= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}, \\
\theta &= \frac{5\pi}{4} + \frac{\pi}{4}i \\
\end{align*}
\]

If \( u = z \), then \( u = \sqrt[4]{3\sqrt{2}} \left( \cos \frac{5\pi}{4} + \frac{\pi}{4}i \right) \\
\]

\[
\begin{align*}
\text{-2} & \text{ not a 4th root of } z \\
\text{-2} & \text{ not a 4th root of } z \\
\text{-1} & \text{ not a 4th root of } z \\
\text{-3} & \text{ not a 4th root of } z \\
\end{align*}
\]

10. (10 points) Find all solutions of \( \cos 4x + \cos 2x = 0 \) in \([0, 2\pi)\). Give exact answers.

\[
\begin{align*}
\text{-2} & \text{ used 1 formula, not } \cos 2x - \sin 2x \\
\text{-1} & \text{ used 1 formula, not } \cos 2x - \sin 2x \\
\text{-1} & \text{ used 1 formula, not } \cos 2x - \sin 2x \\
\text{-2} & \text{ used 1 formula, not } \cos 2x - \sin 2x \\
\end{align*}
\]

\[
\begin{align*}
\cos 4x + \cos 2x &= 0 \\
\cos 2x - \sin 2x + \cos 2x &= 0 \\
\cos^2 2x - (1 - \cos^2 2x) + \cos 2x &= 0 \\
2 \cos^2 2x + \cos 2x - 1 &= 0 \\
(\cos 2x + 1)(\cos 2x + 1) &= 0 \\
\cos 2x &= -1 \\
\cos 2x &= \frac{1}{2} \\
x &= \frac{\pi}{2} + \frac{2\pi}{2} \\
x &= \frac{\pi}{3} + \frac{2\pi}{2} \\
x &= \frac{\pi}{6} + \frac{2\pi}{2} \\
x &= \frac{\pi}{12} + \frac{2\pi}{2} \\
x &= \frac{\pi}{12} + \frac{2\pi}{2} \\
x &= \frac{\pi}{12} + \frac{2\pi}{2} \\
x &= \frac{\pi}{12} + \frac{2\pi}{2} \\
x &= \frac{\pi}{12} + \frac{2\pi}{2} \\
x &= \frac{\pi}{12} + \frac{2\pi}{2} \\
\end{align*}
\]

**BONUS Question** (5 points)

In rectangular coordinates, we are used to using the variables \( x \) and \( y \) in equations. In polar (trigonometric) equations, however, we use the variables \( r \) and \( \theta \). Using a polar graph (the complex plane), graph the following functions:

\[
\begin{align*}
\text{a) } & \quad r = 2 \\
\text{b) } & \quad \theta = 45^\circ
\end{align*}
\]