March 17, 1998 | Math 6 Exam 3 | Name ________________ Instructor ________________

1. Find the exact solutions on $[0,2\pi)$.
(10)

$$\sin 2x \sin x = \cos x$$

2. Let $\sin u = \frac{5}{13}$ where $\frac{\pi}{2} < u < \pi$.
(10) Find the following:

(a) $\cos 2u$

(b) $\cos\left(\frac{u}{2}\right)$

3. Verify $1 + \cos 10y = 2\cos^2 5y$.
(8)

4. Solve the triangle, if possible. If two solutions exist, find both and clearly label.
(12)

(a) $A = 62°, \ a = 10, \ b = 12$

(b) $a = 55, \ b = 25, \ c = 72$

5. The angles of elevation to an airplane from 2 points A and B on level ground are 51° and 68°, respectively.
(10) The points A and B are 2.5 miles apart and the airplane is east of both points in the same vertical plane. Find the altitude of the plane.

6. (a) State Heron's formula.
(2)

(b) Find area of the triangle having:
(4) $a = 5, \ b = 7, \ c = 10$

7. Write the complex number in trig form.
(7)

$$z = -1 + \sqrt{3}\, i$$

8. Multiply or divide and leave the result in trig form.
(10)

(a) $\left[\frac{3}{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right]\left[6\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]$

(b) $\dfrac{2(\cos 120° + i\sin 120°)}{4(\cos 40° + i\sin 40°)}$

9. Use DeMoivre's theorem to find the indicated power of the complex number. Express results in standard form.

(10)

$$\left[2\left(\cos\tfrac{\pi}{2} + i\sin\tfrac{\pi}{2}\right)\right]^8$$

10. Find the cube roots of $8\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$. Leave results in trigonometric form.

(10)

11. Write the product as a sum or difference.

(7)

$$4\sin\tfrac{\pi}{3}\cos\tfrac{5\pi}{6}$$

Sum and Difference Formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$
$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$
$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$
$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$
$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Double Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$
$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Half Angle Formulas

$$\sin \frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}}$$
$$\cos \frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$
$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

Power Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Product to Sum Formulas

$$\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$$
$$\cos u \cos v = \frac{1}{2}\left[\cos(u-v) + \cos(u+v)\right]$$
$$\sin u \cos v = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]$$
$$\cos u \sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]$$

Sum to Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$