Show all work for partial credit. Be neat.

1. Find the equation of the line that is perpendicular to $2x + 3y = 12$ and has the same $y$-intercept.

   $\begin{align*}
   3y &= -2x + 12 \\
   y &= -\frac{2}{3}x + 4
   \end{align*}$

   Slope is $-\frac{2}{3}$

   $y$-intercept

   $\text{Slope} \perp = \frac{3}{2}$

   $1. \quad y = \frac{3}{2}x + 4$

2. Write the equation of the line that passes through the given points.

   a) $(3, -1) \quad (4, -4) \\
   y = m(x - x_1)$

   $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-1)}{4 - 3} = -3$

   $y - (-1) = -3(x - 3)$

   $y + 1 = -3x + 9$

   $y = -3x + 8$

   $2. \quad a) \quad y = -1$

   b) $(2, 4) \quad (4, 4)$

   $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{4 - 2} = 0$

   Parallel line

   $y - 4 = 0(x - 4)$

   $y = 4$

   $b) \quad y = -4x + 12$

   c) $(2, -1) \quad (2, -6)$

   $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - (-1)}{2 - 2}$

   Undefined slope

   $c) \quad x = 2$

3. Find a mathematical expression to model the following:

   z varies directly as the square of x and inversely as y.

   $z = \frac{kx^2}{y}$

   If $z = \frac{3}{2}$ when $x = 3$ and $y = 4$, what is $k$?

   $\frac{3}{2} = \frac{k9}{4}$

   $12 = 18k$

   $k = \frac{2}{3}$
4. Let \( f(x) = 4 - 2x^2; \ g(x) = 2 - x; \ h(x) = \begin{cases} 3 - x^2, & x \geq 0 \\ 3 + 2x, & x < 0 \end{cases} \) Calculate and simplify the following. Show intermediate steps.

\( (g \circ f)(2, 3) = g(3) = 4 - 2 \cdot 3^2 = 4 - 18 = 14 \)

\( h(3) = 3 - 9 = -6 \)

\( h(-3) = 3 + 2(-3) = -3 \)

\( \frac{f(x + 2) - f(x)}{2} \)

\( g(x + 2) = 4 - 2(x + 2)^2 = 4 - 2x^2 - 8x - 8 \)

\( h(-1) = 3 + 2(-1) = 3 - 2 = 1 \)

\( g(-1) = 2 - (-1) = 3 \)

\( h(-1) = 3 \)

\( (g \circ h)(-1) = g(h(-1)) = g(1) = 2 - 1 = 1 \)

5. Find the domain of \( f(x) = \frac{\sqrt{2x + 3}}{x^2 - 5x} \).

\( x^2 - 5x \neq 0 \)

\( x(x - 5) \neq 0 \)

\( x \neq 0, 5 \)

6. Is the given function even or odd?

\( f(x) = -x^4 + 2x^2 - 1 \)

\( f(-x) = -(-x)^4 + 2(-x)^2 - 1 = -x^4 + 2x^2 - 1 \)

\( f(x) = 2x^3 + 3x^2 \)

\( f(-x) = 2(-x)^3 + 3(-x)^2 = -2x^3 + 3x^2 \)

\( f(x) = 4x^3 + 3x \)

\( f(-x) = 4(-x)^3 + 3(-x) = -4x^3 - 3x = -f(x) \)

\( f(x) = \frac{\sqrt{2x + 3}}{x^2 - 5x} \)

\( f(-x) = \frac{\sqrt{2(-x) + 3}}{(-x)^2 - 5(-x)} = \frac{\sqrt{2x + 3}}{x^2 + 5x} \)

6. a) even

b) odd

c) neither
7. Over which interval(s) is the function increasing?
\[ f(x) = 2x^3 + 3x^2 - 12x \]
\[ (-2, 2) \]
\[ (1, \infty) \]

8. Use the graph of \( y = x^3 \) to write an equation for the function \( y = f(x) \) as graphed.
\[ f(x) = (x-2)^3 - 1 \]

9. Given \( f(x) = \sqrt{2x-1} \), state the domain of \( f(x) \). Find \( f^{-1}(x) \).
\[ y = \sqrt{2x-1} \]
\[ x = \frac{y^2 + 1}{2} \]
\[ x = \frac{y^2 + 1}{2} \]
\[ f^{-1}(x) = \frac{x^2 + 1}{2} \]

10. Let \( f(x) = 3 - x \) and \( g(x) = x^3 \). Find \( (g^{-1} \circ f^{-1})(-5) \).
\[ y = 3 - x \]
\[ x = 3 - y \]
\[ g^{-1}(-5) \]
\[ g^{-1}(g^{-1}(-5)) \]
\[ g^{-1}(x) = x \]
\[ 2 \]
11. Given $y = -2x^2 - 4x - 5$.

(a) Write in standard form for a parabola and determine the maximum or minimum value.

Complete the square:

$y = -2(x^2 + 2x) - 5$

$= -2(x^2 + 2x + 1) - 5 + 2$

$= -2(x + 1)^2 - 3$

vertex at $(-1, -3)$ opens down

so max value of $-3$

$\max \text{ at } (-1, -3)$

Equation: $y = -2(x + 1)^2 - 3$

(b) Find the quadratic function that has a maximum point at $(-1, 2)$ and passes through $(0, 1)$.

$y = a(x - h)^2 + k$

$1 = a(0 - (-1))^2 + 2$

$1 = a + 2$

$a = -1$

so $y = -(x + 1)^2 + 2$

11. $\max \text{ of } -3$

12. a) Find the quadratic function that has a maximum point at $(-1, 2)$ and passes through $(0, 1)$.

$y = a(x - h)^2 + k$

$1 = a(0 - (-1))^2 + 2$

$1 = a + 2$

$a = -1$

so $y = -(x + 1)^2 + 2$

b) Find the quadratic function whose graph opens upward and has x-intercepts at $(-4, 0)$ and $(1, 0)$.

$y = a(x - h)^2 + k$

$0 = a(1 + \frac{3}{2})^2 + k$

$k = \frac{4}{25}(x + \frac{3}{2})^2 - 1$

and

$0 = a(-\frac{5}{2})^2 + k$

$k = \frac{4}{25}(x + \frac{3}{2})^2 - 1$

BONUS: Find a relationship between $x$ and $y$ so that $(x, y)$ is equidistant from the two points $(4, -1)$ and $(-2, 3)$.

$m = \frac{3 + 1}{-2 - 4} = \frac{-2}{3}$

$m_{\perp} = \frac{3}{2}$ through $(1, 1)$

so $y - 1 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x - \frac{3}{2} + 1$