Math 4
Exam 2
September 16, 1997

Name Key

Instructor ______
Class Time ______

Show all work for partial credit. Be neat.

1. Find the equation of the line that is perpendicular to $2x+3y=12$ and has the same y-intercept.
(6)

$3y = -2x + 12$

$y = -\frac{2}{3}x + 4$ ← y intercept

slope is $-\frac{2}{3}$

slope ⊥ is $\frac{3}{2}$

$y = \frac{3}{2}x + 4$

1. $y = \frac{3}{2}x + 4$

2. Write the equation of the line that passes through the given points.

(3) a) (3,-1) (-4,-1)

$y = mx+b$ or $y - y_1 = m(x - x_1)$

$m = \frac{-1+1}{3+4} = 0$ 0 slope so horizontal line

2. a) $y = -1$

(3) b) (2,4) (4,-4)

$m = \frac{4+4}{2-4} = \frac{8}{-2} = -4$

$y - 4 = -4(x-2)$

$y - 4 = -4x + 8$

b) $y = -4x + 12$

(3) c) (2,-1) (2,-6)

$m = \frac{-1+6}{2-2}$ ← undef slope so vertical line

c) $x = 2$

3. Find a mathematical expression to model the following:
(8) z varies directly as the square of x and inversely as y.

$z = \frac{kx^2}{y}$

3. $z = \frac{kx^2}{y}$

If $z = \frac{3}{2}$ when $x = 3$ and $y = 4$, what is k?

$\frac{3}{2} = \frac{k9}{4}$

$12 = 18k$

$\frac{12}{18} = k$

$k = \frac{2}{3}$

4. Let $f(x)=4-2x^2$; $g(x)=2-x$; $h(x)=\begin{cases}3-x^2, & x\geq 0\\ 3+2x, & x<0\end{cases}$. Calculate and simplify the following. Show intermediate steps.

(4) a) $(f\circ g)(2.3)$ $g(2.3)=.3$

$f(g(2.3))=f(.3)=4-2(.3^2)=4-.18$

a) 3.82

(4) b) $h(3)-h(-3)$ $=-6-(-3)=-3$

$h(3)=3-9=-6$

$h(-3)=3+2(-3)=-3$

b) -3

(4) c) $\dfrac{f(x+2)-f(x)}{2}$ $\Longrightarrow$ $\dfrac{\cancel{4}-\cancel{2x^2}-8x-8-\cancel{4}+\cancel{2x^2}}{2}$

$f(x+2)=4-2(x+2)^2$

$=4-2x^2-8x-8$ $=-4x-4$

c) $-4x-4$

(4) d) $\left(\frac{g}{h}\right)(-1)$

$g(-1)=2-(-1)=3$ $\frac{g}{h}(-1)=\frac{3}{1}=3$

$h(-1)=3+2(-1)=3-2=1$

d) 3

(4) e) $(g\circ h)(-1)$

$g(h(-1))$ $g(h(-1))=g(1)=2-1=1$

$h(-1)=1$

e) 1

5. Find the domain of $f(x)=\dfrac{\sqrt{2x+3}}{x^2-5x}$.

(5) $x^2-5x\neq 0$ also $\sqrt{2x+3}\geq 0$ $x\geq\frac{-3}{2}$

$x(x-5)\neq 0$ $2x+3\geq 0$

$x\neq 0, 5$

5. $[-\frac{3}{2},0)\cup(0,5)\cup(5,\infty)$

6. Is the given function even or odd?

(3) a) $f(x)=-x^4+2x^2-1$

$f(-x)=-(-x)^4+2(-x)^2-1$

$=-x^4+2x^2-1=f(x)$

6. a) even

(3) b) $f(x)=2x^3+3x^2$

$f(-x)=2(-x)^3+3(-x)^2$

$=-2x^3+3x^2$

b) neither

(3) c) $f(x)=4x^3+3x$

$f(-x)=4(-x)^3+3(-x)$

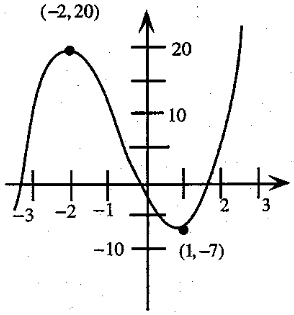
$=-4x^3-3x=-(4x^3+3x)$

$=-f(x)$

c) odd

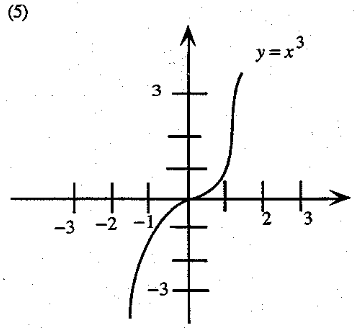
7. Over which interval(s) is the function increasing?
(5)

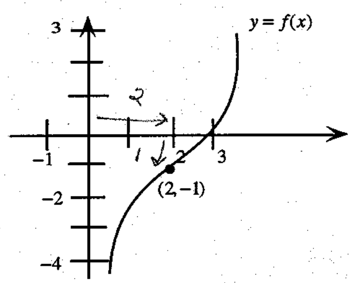
$$f(x) = 2x^3 + 3x^2 - 12x$$



7. $(-\infty, -2), (1, \infty)$

8. Use the graph of $y = x^3$ to write an equation for the function $y = f(x)$ as graphed.
(5)





Shifted 2 units right so $(x-2)^3$ and 1 unit down

$f(x) = (x-2)^3 - 1$

9. Given $f(x) = \sqrt{2x-1}$, state the domain of $f(x)$. Find $f^{-1}(x)$.
(7)

$2x - 1 \geq 0$

$x \geq \frac{1}{2}$

$y = \sqrt{2x-1}$

$y^2 = 2x - 1$

$y^2 + 1 = 2x$

$\frac{y^2+1}{2} = x$

$\frac{x^2+1}{2} = f^{-1}(x)$

9. $x \geq \frac{1}{2}$

$f^{-1}(x) = \frac{x^2+1}{2}$

10. Let $f(x) = 3 - x$ and $g(x) = x^3$. Find $(g^{-1} \circ f^{-1})(-5)$.
(8)

$y = 3 - x$

or exchange x, y first

$x = 3 - y$

$y = 3 - x = f^{-1}$

$f^{-1}(-5) = 8$

$g^{-1}(f^{-1}(-5))$

$g(x) = y = x^3$

$x = y^3$

$\sqrt[3]{x} = y = g^{-1}(x)$

$g^{-1}(8) = \sqrt[3]{8} = 2$

10. 2

11. Given $y = -2x^2 - 4x - 5$.

(8) Write in standard form for a parabola and determine the maximum or minimum value.

Complete the square

$y = -2(x^2 + 2x \quad) - 5$

$= -2(x^2 + 2x + 1) - 5 + 2$

$= -2(x+1)^2 - 3$

vertex at $(-1,-3)$ opens down

so max value of -3

11. max of -3

equation: $y = -2(x+1)^2 - 3$

12. a) Find the quadratic function that has a maximum point at (-1,2) and passes through (0,1).

(5) (h,k) (x,y)

$y = a(x-h)^2 + k$

$1 = a(0-(-1))^2 + 2$

$1 = a + 2$

$-1 = a$ so $y = -1(x+1)^2 + 2$

a) $y = -1(x+1)^2 + 2$

b) Find the quadratic function whose graph opens upward and has x-intercepts at (-4,0) and(1,0).

(5) $y = a(\overset{x}{-4} + \overset{h}{\tfrac{3}{2}})^2 + k$ (choose) let $h = -\frac{3}{2}$, $k = -1$

$0 = a(1 + \frac{3}{2})^2 + k$ so $y = \frac{4}{25}(x + \frac{3}{2}) - 1$

so $0 = (-\frac{5}{2})^2 a + k$

and $0 = a(\overset{x}{1} + \frac{3}{2})^2 - 1$

$\frac{4}{25} = a$

b) $y = \frac{4}{25}(x + \frac{3}{2})^2 - 1$

not unique

Bonus: Find a relationship between x and y so that (x,y) is equidistant from the two points (4,-1) and (-2,3).

(5) $m = \dfrac{3+1}{-2-4} = -\dfrac{2}{3}$ mdpt $\left(\dfrac{4+(-2)}{2}, \dfrac{-1+3}{2}\right) \Rightarrow (1,1)$

$m_\perp = \frac{3}{2}$ thru $\overset{\text{mdpt}}{(1,1)}$

$y = \frac{3}{2}x - \frac{1}{2}$

so $y - 1 = \frac{3}{2}(x-1)$

$y = \frac{3}{2}x - \frac{3}{2} + 1$