Math 4
Exam 2
September 15, 1998

Name KEY

Instructor _____
Class Time _____

Show all work for partial credit.

1. (10 pts) Given that $p_1 = (5,-2)$ and $p_2 = (-1,6)$ find each of the following:

   (a) The distance from p_1 to p_2. $d = \sqrt{(5+1)^2 + (-2-6)^2} = \sqrt{36+64} = 10$

   (b) The coordinates of the midpoint of the line segment connecting p_1 to p_2. $(2,2)$

   $x = \frac{5-1}{2} = 2, \quad y = \frac{-2+6}{2} = 2$

   (c) The slope-intercept form of the equation of the line passing through p_1 and p_2.

   $m = \frac{6+2}{-1-5} = \frac{8}{-6} = -\frac{4}{3}$

   $y + 2 = m(x-5)$

   $= -\frac{4}{3}(x-5)$

   $y = -\frac{4}{3}x + \frac{20}{3} - \frac{6}{3}$

   $y = -\frac{4}{3}x + \frac{14}{3}$

   (a) 10

   (b) (2,2)

   (c) $-4/3\ x + 14/3$

2. (6 pts) Given that $f(x) = 5x^2 - x$, find $f(1)$ and $f(x-1)$.

   $f(1) = 5 - 1 = 4$

   $f(x-1) = 5(x-1)^2 - (x-1)$

   $= 5x^2 - 10x + 5 - x + 1$

   $= 5x^2 - 11x + 6$

   $f(1) =$ 4

   $f(x-1) =$ $5x^2 - 11x + 6$

3. (6 pts) Given that $f(x) = \begin{cases} 4+5x & \text{if } x \le 1 \\ 1-x^2 & \text{if } x > 1 \end{cases}$, find $f(-2)$ and $f(3)$.

   $f(-2) = 4 - 10 = -6$

   $f(3) = 1 - 9 = -8$

   $f(-2) =$ -6

   $f(3) =$ -8

4. (10 pts) Find the center and the radius of the following circle. Then sketch the graph of the circle.

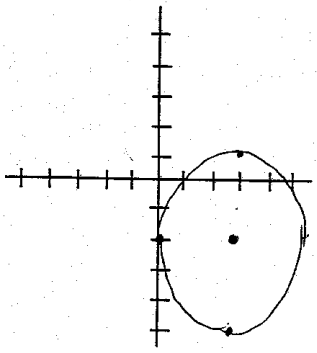
$$x^2 + y^2 - 6x + 4y = -4$$

$(x^2 - 6x + 9) + (y^2 + 4y + 4) = -4 + 9 + 4$

$(x-3)^2 + (y+2)^2 = 9$

center $(3, -2)$

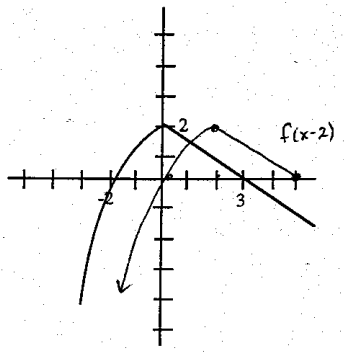
radius 3

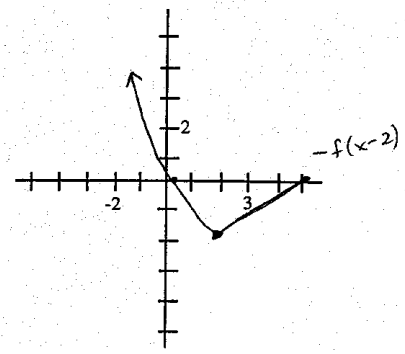


center = $(3, -2)$

radius = 3

5. (6 pts) Given that $y = f(x)$ has the graph below, draw the graph of $y = -f(x-2)$.





6. (6 pts) Find the inverse, if it exists, of the function

$$f(x) = x^2 - 4, \quad x \geq 0$$

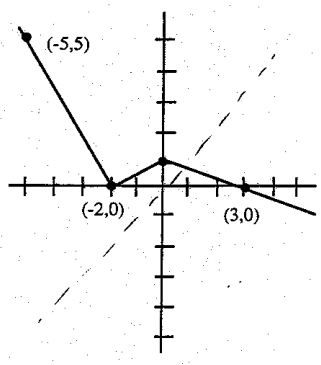
$x = y^2 - 4$

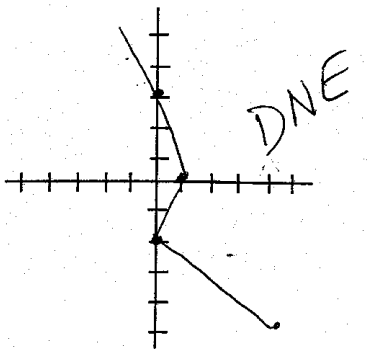
$x + 4 = y^2$

$y = \sqrt{x+4} \quad y \geq 0$

$f^{-1}(x) = \sqrt{x+4}$

7. (6 pts) Given that $y = f(x)$ has the graph below, sketch the graph of $f^{-1}(x)$.





8. (6 pts) Determine the types of symmetry the following equation has (if any). That is, does it have symmetry with respect to the x-axis, y-axis and/or the origin? Support your answer.

$$y = \pm\sqrt{5x^2+4}$$

(a) x-axis

If (x,y) is on graph, so is $(x,-y)$ b/c of $\pm$ sign

(a) yes

(b) y-axis

$y = \pm\sqrt{5(-x)^2+4}$ if (x,y) on graph
$= \pm\sqrt{5x^2+4}$ so is $(-x,y)$

(b) yes

(c) origin

$-y = \pm\sqrt{5(-x)^2+4}$ if (x,y) on graph, so is $(-x,-y)$
$y = \mp\sqrt{5x^2+4}$

(c) yes

9. (6 pts) Find the x and y-intercepts of the following equation. If there does not exist an intercept say so. Note that intercepts are points.

$$y = \sqrt{x^2+4}$$

y-int: $x=0$, $y=\sqrt{4}$ $(0,2)$

x-int: $y=0$, $0=\sqrt{x^2+4}$

$0=x^2+4$ no x-int

$-4=x^2$

x-intercepts none

y-intercepts $(0,2)$

10. (15 pts) Given that $f(x)=x+5$, $g(x)=x^2+1$ and $h(x)=\sqrt{4-x^2}$, evaluate each of the following.

(a) $g(2)-f(2)=$ $(2^2+1)-(2+5)=5-7=-2$

(b) $\frac{f(4)}{g(4)}=$ $\frac{9}{17}$

(c) $g(3)*f(3)=$ $10\cdot 8=80$

(d) $g(f(x))=$ $g(x+5)=(x+5)^2+1=x^2+10x+26$

(e) $h(f(-4))=$ $h(-4+5)=h(1)=\sqrt{4-1}=\sqrt{3}$

11. (12 pts) If $f(x)=-x^2-4x+1$, identify the vertex and the equation of the axis of symmetry.

vertex at $x=\frac{-b}{2a}=\frac{4}{-2}=-2$

$y=-(-2)^2-(4\cdot -2)+1$

$=-4+8+1$

$=5$

vertex $(-2,5)$

axis $x=-2$

$f(x)=-(x^2+4x+4-4-1)$

$=-(x+2)^2+5$

vertex $(-2,5)$

axis $x=-2$

vertex $(-2,5)$

axis of symmetry $x=-2$

12. (11 pts) Find a mathematical model representing the following statement. In addition, determine the constant of proportionality.

- z is jointly proportional to x and y. ($z=10$ when $x=20$ and $y=4$)

$z=kxy$

$10=k(20)(4)=80k$

$k=1/8$

Model: $z=kxy$

constant: $k=1/8$