Name KEY
Instructor_____

Math 6, Exam 2
Monday, November 24, 1997

Examination Rules:

1. If you have a question, please raise your hand.
2. All work must be shown.
3. Use exact values, unless asked to do otherwise.
4. Please circle your final answer.

DO NOT TURN PAGE UNTIL TOLD TO DO SO.

10/10

7. (5 pts.) If $\sin(-x)=-\frac{2}{3}$ and $\tan x=-\frac{2}{\sqrt{5}}$, find the values of the other trigonometric functions.

1 $\sin x = \frac{2}{3}$

$\tan x = \frac{\sin x}{\cos x} = \frac{2/3}{\cos x} = \frac{-2}{\sqrt{5}}$

2 $\cos x = \frac{2}{3} \cdot \frac{-\sqrt{5}}{2} = -\frac{\sqrt{5}}{3}$

1.5

$\csc x = 3/2$

$\cot x = \frac{-\sqrt{5}}{2}$

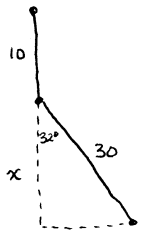
$\sec x = -\frac{3}{\sqrt{5}}$

1.5

1 1 } 2

9/10

8. (10 pts.) A ship leaves port traveling due south. After 30 minutes, the ship must change course to S 32° E to avoid a storm. If the ship maintains a speed of 20 knots, how far south will the ship have traveled 2 hours after leaving port?



$30 \text{ min} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{20 \text{ n.m.}}{1 \text{ hr}} = 10 \text{ n.m}$

total 2 hrs, so going SE for 90 min.

$90 \text{ min} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{20 \text{ nm}}{1 \text{ hr}} = 30 \text{ n.m.}$

$\cos 32° = \frac{x}{30}$

$x = 30 \cos 32°$

$x \approx 25.44 \text{ n.m}$

total distance south $\approx 10 + 25.44 = 35.44 \text{ n.m.}$

7.3/5-13

1. (5 pts.) Use the trigonometric substitution $x = 3 \sin\theta$ to write

$$\sqrt{27-3x^2}$$

as a trigonometric function of θ, where $0 < \theta < \pi/2$.

$$\begin{aligned}\sqrt{27-3x^2} &= \sqrt{27-3(3\sin\theta)^2} \\ &= \sqrt{27-27\sin^2\theta} \\ &= 3\sqrt{3}\sqrt{1-\sin^2\theta} \\ &= 3\sqrt{3}\,|\cos\theta| \\ &= 3\sqrt{3}\cos\theta\end{aligned}$$

7.2/5-12

2. (12 pts.) Matching:

$\dfrac{1+\sec x}{\sin x+\tan x}$ = C

$\dfrac{\cos(-x)}{1+\sin(-x)}$ = B

$\dfrac{\cot^2 x}{\csc x+1}$ = A

A) csc x - 1

B) sec x + tan x

C) csc x

D) 1 + cot(-x)

E) csc x + sin x

3. (2 pts each) Clearly indicate whether the following statements are **true** or **false**:

F sec x cos y = 1

F 1/(5 cos θ)=5 sec θ

F $\cos(\frac{\pi}{2}-x)=\csc x$

T A point moving in simple harmonic motion described by the equation $d=2\sin(4\pi x)$ has a frequency of 2 cycles/unit time.

4. (10 pts. each) Verify the following identities:

A) $\dfrac{\sin^3 x + \cos^3 x}{\sin x \cos x} = \sin x \tan x + \cos x \cot x$

$$\frac{\sin^3 x + \cos^3 x}{\sin x \cos x} = \frac{\sin^3 x}{\sin x \cos x} + \frac{\cos^3 x}{\sin x \cos x}$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\sin x}$$

$$= \sin x \tan x + \cos x \cot x$$

B) $\dfrac{1-\cos x}{\cos x} = \dfrac{\tan^2 x}{\sec x + 1}$

$$\frac{1-\cos x}{\cos x} = \frac{1}{\cos x} - 1$$

$$= \sec x - 1 \cdot \frac{(\sec x + 1)}{\sec x + 1}$$

$$= \frac{\sec^2 x - 1}{\sec x + 1}$$

$$= \frac{\tan^2 x}{\sec x + 1}$$

C) $\dfrac{\csc^2 \theta - 1}{1 - \sin\theta} = \dfrac{1 + \sin\theta}{\sin^2 \theta}$

$$\frac{\csc^2\theta - 1}{1 - \sin\theta} = \frac{\frac{1 - \sin^2\theta}{\sin^2\theta}}{\frac{1 - \sin\theta}{1}} = \frac{1 - \sin^2\theta}{\sin^2\theta(1 - \sin\theta)}$$

$$= \frac{(1 - \sin\theta)(1 + \sin\theta)}{\sin^2\theta(1 - \sin\theta)}$$

$$= \frac{1 + \sin\theta}{\sin^2\theta}$$

7.3/5.13

5. (10 pts.) Find **all** solutions of the equation $\sin x \cos x - \cos x = 0$. Use exact values.

$\cos x(\sin x - 1) = 0$

$\cos x = 0$	$\sin x = 1$ 4
$x = \pi/2 + n\pi$	$x = \pi/2 + 2n\pi$
3	3

$x = \pi/2 + n\pi$

7.3/5.13

6. (10 pts. each) Find all solutions of the following equations in the interval $[0, 2\pi)$:

A) $2\sin^2(3x) = 1$

just got $\pi/12$, didn't put $\pm$, no picture −6

$\sin 3x = \frac{1}{\sqrt{2}}$

$3x = \pi/4$

$x = \pi/12$

$\sin 3x = \pm \frac{1}{\sqrt{2}}$

$3x = \pi/4 + \frac{n\pi}{2}$

$x = \pi/12 + \frac{2n\pi}{12}$

$x = \pi/12, 3\pi/12, 5\pi/12, 7\pi/12, 9\pi/12, 11\pi/12, 13\pi/12, 15\pi/12, 17\pi/12, 19\pi/12, 21\pi/12, 23\pi/12$

−2 didn't list out far
−3 chose before div. by 3
−3 ±

$3x = \pm \frac{\pi}{4} + 2n\pi$ −3

B) $\dfrac{1+\sin x}{\cos x} + \dfrac{\cos x}{1+\sin x} = -2$

−8 if couldn't reduce

$\dfrac{(1+\sin x)^2 + \cos^2 x}{\cos x(1+\sin x)} = -2$

$\dfrac{1 + 2\sin x + 1}{\cos x(1+\sin x)} = -2$

$\dfrac{2}{\cos x} = -2$

$\cos x = -1$

$x = \pi$

−2 if $\sec x = 1$ instead of −1