Math 6, Exam III

Thursday, December 11, 1997

NAME KEY

Instructor, time Steph

Do not turn this page until told to do so.

You are free to use the formula sheet on the last page.

1. (14 points) By showing all possible solutions or explaining why there are none, solve the triangles with sides and angles as follows:

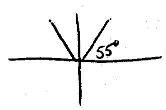
a) $A = 120°, a = 12, b = 15.$

No solutions, A is largest ∡ and a is not longest side.

b) $A = 35°, a = 7, b = 10.$

$\frac{\sin B}{10} = \frac{\sin 35°}{7}$

$\sin B = \frac{10}{7} \sin 35° \approx .819$



$B \approx 55°$ or $B \approx 125°$

$C \approx 90°$ $\quad C \approx 20°$

$c = \frac{\sin C \cdot 7}{\sin 35°}$

$c \approx 12.2$ $\quad c \approx 4.2$

2. (12 points) Find the exact values of

a) $\cos\frac{\pi}{12}$. $= \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\pi/3 - \pi/4\right) = \cos\pi/3 \cos\pi/4 + \sin\pi/3 \sin\pi/4$

$= \frac{1}{2\sqrt2} + \frac{\sqrt3}{2\sqrt2} = \frac{1+\sqrt3}{2\sqrt2} = \frac{\sqrt2 + \sqrt6}{4}$

b) $\csc(-75°)$. $= -\csc 75° = \frac{-1}{\sin 75°} = \frac{-1}{\sin(45° + 30°)} = \frac{-1}{\sin 45° \cos 30° + \sin 30° \cos 45°}$

$= \frac{-1}{\frac{\sqrt3}{2\sqrt2} + \frac{1}{2\sqrt2}} = \frac{-1}{\frac{1+\sqrt3}{2\sqrt2}} = \frac{-2\sqrt2}{1+\sqrt3} = \frac{-4}{\sqrt6 + \sqrt2}$

$\frac{\sqrt6 - \sqrt2}{4} \stackrel{?}{=} \frac{1+\sqrt3}{2\sqrt2}\sqrt2 \quad \frac{-2}{\times\sqrt2 - 2\sqrt6}$

3. (10 points) Verify that $\sin(n\pi + \theta) = (-1)^n \sin\theta$ for all integers n.

$$\sin(n\pi + \theta) = \sin n\pi \cos\theta + \cos n\pi \sin\theta$$
$$= 0 + \cos n\pi \sin\theta$$
$$= (-1)^n \sin\theta$$

4. (10 points) Calculate the following and write your answer in standard form:

a) $$\frac{3\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)}{6\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)} = \frac{1}{2}\left(\cos \pi/3 + i\sin \pi/3\right)$$
$$= \frac{1}{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$
$$= \frac{1}{4} + \frac{\sqrt{3}}{4} i$$

b) $$\left[2\left(\cos\frac{4\pi}{15} + i\sin\frac{4\pi}{15}\right)\right]^5 = 32\left(\cos\frac{4\pi}{3} + i\sin 4\pi/3\right)$$
$$= 32\left(\frac{-1}{2} - \frac{\sqrt{3}}{2} i\right)$$
$$= -16 - 16\sqrt{3}\, i$$

5. (12 points) Find the areas of the triangles with angles and sides as follows:

a) $a = 12, b = 15, c = 9$.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \qquad s = \frac{a+b+c}{2} = \frac{36}{2} = 18$$

$$= \sqrt{(18)(6)(3)(9)}$$

$$= \sqrt{2916}$$

$$= 54$$

b) $A = 130°, b = 62, c = 20$.

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2}(62)(20) \sin 130°$$

$$= 474.95$$

6. (10 points) Find all solutions of $\cos 4x - 7\cos 2x = 8$ in $[0, 2\pi)$.

$$2\cos^2 2x - 1 - 7\cos 2x - 8 = 0$$

$$(2\cos 2x - 9)(\cos 2x + 1) = 0$$

$\cos 2x = 9/2$ no solution

$\cos 2x = -1$

$$2x = \pi + 2n\pi$$

$$x = \frac{\pi}{2} + n\pi$$

In $[0, 2\pi)$, $x = \pi/2, 3\pi/2$

got all in 2x, [redacted] no further −6

not all in 2x −8

7. (5 points) Write $5+12i$ in trigonometric form.

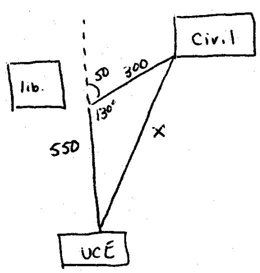
$r = \sqrt{25+144} = 13$

$\tan\theta = 12/5$

$\theta \approx 67.4°$

$5+12i = 13\,(\cos 67.4° + i \sin 67.4°)$

8. (7 points) To get from University Center - East to Civil Engineering, you can walk 550 ft North to the library, turn to a bearing of N 50° E, and then walk 300 ft to Civil Engineering. (Assume these are all straight line paths). How far is it in a straight line between the two buildings?



$x^2 = 550^2 + 300^2 - 2(550)(300)\cos 130°$

$x \approx 777.57$ ft

9. **(10 points) Find all fourth roots of i.**

$i = \cos \pi/2 + i \sin \pi/2$

$u^4 = i$, so

$u = \sqrt[4]{1}\left(\cos \frac{\pi/2 + 2k\pi}{n} + i \sin \frac{\pi/2 + 2k\pi}{n}\right)$

$k=0 \quad u = \cos \pi/8 + i \sin \pi/8 \approx .9239 + .3827i$

$k=1 \quad u = \cos 5\pi/8 + i \sin 5\pi/8 \approx -.3827 + .9239i$

$k=2 \quad u = \cos 9\pi/8 + i \sin 9\pi/8 \approx -.9239 - .3827i$

$k=3 \quad u = \cos 13\pi/8 + i \sin 13\pi/8 \approx .3827 - .9239i$

10. **(10 points) Find all solutions of $\sin\frac{x}{2} + \cos x = 1$.**

can't square both sides

$\pm\sqrt{\frac{1-\cos x}{2}} + \cos x = 1$

$\frac{1-\cos x}{2} = 1 - 2\cos x + \cos^2 x$

$1 - \cos x = 2 - 4\cos x + 2\cos^2 x$

$2\cos^2 x - 3\cos x + 1 = 0$ ← to here −5

$(2\cos x - 1)(\cos x - 1) = 0$

$2 \neq -2$

but $4 = 4$

$\sin\frac{x}{2} + \cancel{1} - 2\sin^2\frac{x}{2} = \cancel{1}$

$\sin\frac{x}{2}\left(1 - 2\sin\frac{x}{2}\right) = 0$

$\cos x = 1/2 \qquad \cos x = \phi$

$x = \begin{cases} \pi/3 + 2n\pi \\ 5\pi/3 + 2n\pi \end{cases} \qquad x = \begin{cases} 0 + 2n\pi \\ \end{cases}$

$\sin\frac{x}{2} = 0 \qquad \sin\frac{x}{2} = \frac{1}{2}$

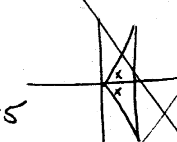
$\frac{x}{2} = n\pi \qquad \frac{x}{2} = \begin{cases} \frac{\pi}{6} + 2n\pi \\ \frac{5\pi}{6} + 2n\pi \end{cases}$

$x = 2n\pi \qquad x = \begin{cases} \pi/3 + 4n\pi \\ 5\pi/3 + 4n\pi \end{cases} \qquad \pi/3 + 2\pi \qquad \frac{7\pi}{3}$

$\sin\frac{7\pi}{6} + \cos\frac{7\pi}{3}$

$-\frac{1}{2} - \frac{1}{2} = -1$

$\cancel{\cos x = \frac{3 \pm \sqrt{9-4}}{2}} \approx 2.618$ or $.38197$



$x \approx \begin{cases} 67.54° + 360°n \\ -67.54° + 360°n \end{cases}$