

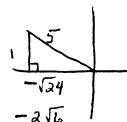
You have 50 minutes to complete this test. You must *show all work* to receive full credit. Each question is worth the indicated value, for a total of 100 points possible. You may also earn 5 bonus points from the bonus problem. If you have any questions, please come to the front and ask.

- 6/6
7.1
5/10 1. (10 points) Complete this chart:

Function	Domain	Range
$y = \sin x$	\mathbb{R}	$-1 \leq y \leq 1$
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\pi/2 \leq y \leq \pi/2$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	\mathbb{R}	$-\pi/2 < y < \pi/2$

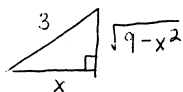
- 7.1
5/10 2. (5 points) If $\csc x = 5$ and $\cos x < 0$, find the ^{exact} values of all 6 trigonometric functions of x .

$$\begin{aligned} \sin x &= 1/5 & \csc x &= 5 \\ \cos x &= -\frac{2\sqrt{6}}{5} & \sec x &= -\frac{5}{2\sqrt{6}} \\ \tan x &= -\frac{1}{2\sqrt{6}} & \cot x &= -2\sqrt{6} \end{aligned}$$



- 6/6
5/10 3. (5 points) Rewrite $\tan\left(\cos^{-1}\frac{x}{3}\right)$ as an algebraic expression. Drawing a right triangle will help. Your answer should contain no trigonometric functions.

$$\tan \theta, \text{ where } \theta \text{ has } \cos \theta = \frac{x}{3}$$



$$\tan\left(\cos^{-1}\frac{x}{3}\right) = \frac{\sqrt{9-x^2}}{x} \quad \left(\text{or } -\frac{\sqrt{9-x^2}}{x}\right)$$

7.2
5.4

(20 points) Verify these identities:

a) $\frac{\cos x \csc x}{\cot^2 x} = \tan x$

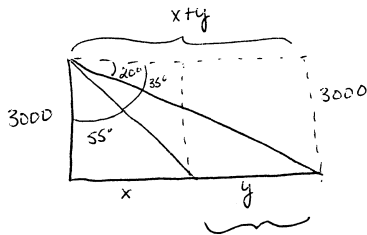
$$\begin{aligned} \frac{\cos x \csc x}{\cot^2 x} &= \cos x \cdot \frac{1}{\sin x} \cdot \tan^2 x \\ &= \frac{\cos x}{\sin x} \cdot \frac{\sin^2 x}{\cos^2 x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \end{aligned}$$

b) $\cos\left(x - \frac{\pi}{2}\right) \sec x - \frac{1}{\cos x \sec\left(\frac{\pi}{2} - x\right)} = 0$

$$\begin{aligned} \cos\left(x - \frac{\pi}{2}\right) \sec x - \frac{1}{\cos x \sec\left(\frac{\pi}{2} - x\right)} &= \cos\left(-\left(\frac{\pi}{2} - x\right)\right) \frac{1}{\cos x} - \frac{1}{\cos x \csc x} \\ &= \cos\left(\frac{\pi}{2} - x\right) \cdot \frac{1}{\cos x} - \frac{\sin x}{\cos x} \\ &= \frac{\sin x}{\cos x} - \frac{\sin x}{\cos x} \\ &= 0 \end{aligned}$$

6.7
5

(10 points) An airplane pilot is following the direction of a highway, at an altitude of 3000 ft. He sees two trucks on the highway ahead. The angle of depression to the farther truck is 20° and to the closer is 35° . How far apart are the trucks?



$$\tan 20^\circ = \frac{3000}{x+y}$$

$$x+y = \frac{3000}{\tan 20^\circ} \approx 8242.43$$

$$\tan 35^\circ = \frac{x}{3000}$$

$$x = 3000 \tan 35^\circ \approx 4284.44$$

$$y = \text{distance between trucks}$$

$$\approx 8242.43 - 4284.44$$

$$\approx 3957.99 \text{ ft}$$

7/2
5

6. (12 points) Simplify and match:

$$\frac{\csc x}{\tan x + \cot x} = \underline{e}$$

a) $-\cos^2 x$

$$\frac{\frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{1}{\sin x}}{\frac{1}{\sin x \cos x}} = \cos x$$

b) $\cos^2 x$

$$\sin\left(\frac{\pi}{2} - x\right) \cos(-x) = \underline{b}$$

$\cos x \cos x$

c) $\sec^2 x$

d) $2 \cos x$

$$\frac{\sin^2 x}{\sec^2 x - 1} = \underline{b}$$

e) $\cos x$

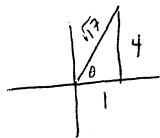
$$\frac{\sin^2 x}{\tan^2 x} = \frac{\sin^2 x}{\sin^2 x / \cos^2 x}$$

f) $-\sin x \cos x$

7/2
5

7. (10 points) Find the exact value of $\cos(\tan^{-1} 4)$.

$$\tan \theta = 4$$



$$\cos(\tan^{-1} 4) = \frac{1}{\sqrt{17}}$$

8. (8 points) Circle True or False -- Mark answers CLEARLY.

- a) $\cos \theta = \sqrt{1 - \sin^2 \theta}$ TRUE FALSE
- b) $\sin^{-1}(\sin x) = x$ TRUE FALSE
- c) An object in harmonic motion with displacement $d = 3 \cos \pi x$ has a frequency of 0.5 cycles per unit of time. TRUE FALSE
- d) $3 \sin x = \frac{1}{3 \csc x}$ TRUE FALSE

9.

(10 points) Find all solutions of $2 \sin^2 x + 3 = 5 \sin x$.

$$2 \sin^2 x - 5 \sin x + 3 = 0$$

$$(2 \sin x - 3)(\sin x - 1) = 0$$

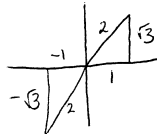
$$\sin x = 3/2 \quad \sin x = 1$$

NO

$$x = \pi/2 + 2n\pi$$

10.

(10 points) Find all solutions of $\tan 3t = \sqrt{3}$ in the interval $[0, 2\pi)$.



$$3t = \begin{cases} \pi/3 + 2n\pi \\ 4\pi/3 + 2n\pi \end{cases}$$

$$3t = \pi/3 + n\pi$$

$$t = \pi/9 + n\pi/3$$

$$= \pi/9, 4\pi/9, 7\pi/9, 10\pi/9, 13\pi/9, 16\pi/9$$

BONUS Question (5 points)

What bridge contains enough wire to wrap around the world three times? Golden Gate

What is written on John Wayne's gravestone? nothing

What are the dots on dominoes called? piPs