Math VI, Exam II

Wednesday, February 25th, 1998

Name : _____

1. (8 points each) Verify the following identities;

(a) $\sin^2 x - \sin^4 x = \cos^2 x - \cos^4 x$

(b) $\dfrac{\cos(-x)}{1 + \sin(-x)} = \sec x + \tan x$

(c) $\dfrac{1}{\cot x + 1} + \dfrac{1}{\tan x + 1} = 1$

2. (6 points) *Explain* why the following is not an identity and give one value of x for which the equation does not hold.

$$\sin x = \sqrt{1 - \cos^2 x}$$

3. (6 points each) Find the *exact* value of the following;

(a) $\cos 57^\circ \cos 12^\circ + \sin 57^\circ \sin 12^\circ$

(b) $\sin 15^\circ$

4. (8 points) Verify that $\cos(\pi - x) + \sin(\pi/2 + x) = 0$

5. (10 points each) Find **all** solutions of the following three equations;

(a) $\sin(3x) = -\sqrt{3} / 2$

(b) $\sin^2 x = 3\cos^2 x$

(c) $\sin(x + \pi/3) + \sin(x - \pi/3) = 1$

6. (10 points) Rewrite $1 / (1 + \sin x)$ so that it is not in fractional form.

7. (5 points each) Use the given substitutions to express the algebraic expressions in trigonometric form.

(a) $\sqrt{16 - 4x^2}$, $x = 2\sin\theta$, $-\pi < \theta < 0$

(6) $\sqrt{100 + x^2}$, $x = 10\tan\theta$, $0 < \theta < \frac{\pi}{2}$